A Credibility-Based CVaR Model for Cryptocurrency Portfolio Optimization under Partial Information

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ABSTRACT

The cryptocurrency market presents profitable yet highly uncertain investment opportunities, marked by rapid expansion and strong volatility. Conventional risk management approaches, built on probabilistic models and past data, often fail to capture the distinctive behavior and unpredictability of this market. To address these issues, this study introduces a portfolio optimization framework designed specifically for cryptocurrencies, based on a credibilistic Conditional Value at Risk (CVaR) model. CVaR is applied as the main risk metric because it targets downside risk and extreme losses, making it suitable for handling sharp downturns common in volatile digital assets. The model integrates credibility theory and trapezoidal fuzzy variables, enabling better representation of uncertainty and market instability. In contrast with traditional probability-based methods, this approach allows for more flexible and accurate risk control. The framework also accounts for real-world restrictions such as cardinality and floor–ceiling limits, which encourage diversification and reflect costs and regulatory requirements. Results from empirical testing confirm the ability of the model to build diversified portfolios that maintain a balance between risk and return. Overall, this work advances portfolio optimization for digital assets by applying advanced methods that provide investors with a practical and effective tool for managing risk in a complex and rapidly changing financial environment.

Keywords: Portfolio optimization, Cryptocurrency market, Credibility theory, Fuzzy uncertainty, Conditional Value at Risk (CVaR), Practical constraints

1. INTRODUCTION

Portfolio optimization is one of the most dynamic and evolving topics in modern financial theory [1], with its foundations tracing back to the pioneering work of Harry Markowitz [2] in 1952. In his seminal paper published that year, Markowitz put forward the mean-variance model for portfolio selection, framing it as a bi-criteria optimization problem with a tradeoff between risk and profit. This contribution laid the foundation for modern portfolio theory, which has since become a fundamental aspect of investment management and financial decision-making. The core premise of portfolio optimization is the recognition that risk and return are inextricably linked, and that by carefully balancing the composition of an investment portfolio, investors can potentially enhance their risk-adjusted returns. Portfolio selection problems have, consequently, attracted significant attention in the realm of risk management, with variance emerging as a prominent tool for quantifying and managing investment risk.

In the pursuit of more sophisticated risk measures, researchers have developed a variety of alternative approaches, with Value at Risk (VaR) [3] and Conditional Value at Risk (CVaR) [4] garnering considerable attention and adoption in the financial industry [5]. However, it is important to note that uncertainty remains a prevalent aspect of the capital markets, as much of the information used to make investment decisions is inherently uncertain, imprecise, or incomplete [6]. To address the challenges posed by uncertainty, researchers have explored alternative approaches, such as the fuzzy set theory introduced by Zadeh [7]. One of the more recent developments in this domain is credibility theory, which was introduced by Liu [8] and further developed in Liu's [9] work. In the context of fuzzy decision systems, portfolio optimization problems have been studied using credibility-based models, with researchers proposing innovative credibility measures for modeling asset returns.

This paper aims to address this gap by proposing a novel portfolio allocation model that incorporates the cryptocurrency market within a credibilistic CVaR optimization framework. CVaR is selected as the primary risk measure due to its focus on downside risk, specifically targeting extreme losses, which makes it particularly effective for managing the heightened risk of sharp downturns in highly volatile markets like cryptocurrencies. The model incorporates credibility theory and trapezoidal fuzzy variables to more accurately reflect the uncertainty and volatility inherent in digital assets. Additionally, the approach includes practical constraints such as cardinality, and floor and ceiling constraints, ensuring the portfolio remains diversified, balanced, and aligned with real-world factors like transaction costs and regulatory requirements. By leveraging credibility theory to capture the uncertainty in asset returns and incorporating practical investment constraints, the proposed model allows investors to build well-diversified portfolios that effectively balance risk and return in the cryptocurrency market. This research represents a significant step forward in the application of advanced portfolio optimization techniques to the dynamic and rapidly changing landscape of digital assets.

The remainder of this paper is organized as follows: Section 2 presents a comprehensive literature review. Section 3 delves into the key concepts and definitions of fuzzy numbers and credibility theory. In Section 4, the paper provides an in-depth description of the suggested portfolio selection model. Section 5 presents the empirical results and computational analysis of the proposed portfolio optimization model. Finally, Section 6 encapsulates the key conclusions drawn from the study and explores potential avenues for further development of the proposed approach.

2. LITERATURE REVIEW

2.1. Related studies

Researchers and financial analysts have increasingly focused on developing various models and strategies to manage risk and enhance returns in cryptocurrency investments. Bowala and Singh [10] optimized the portfolio risk associated with cryptocurrencies by employing data-driven risk measures that account for the high skewness and kurtosis of cryptocurrency returns. The authors argue that traditional risk measures, which often assume normality in asset returns, tend to underestimate the risks associated with cryptocurrency investments due to their failure to account for the non-normal distribution of returns. To address this issue, the paper introduces a novel approach to risk forecasting that leverages high-frequency data to better capture the volatility and risk dynamics of cryptocurrencies. The paper's objective is to demonstrate that this data-driven method is superior to traditional methods in optimizing cryptocurrency portfolios.

Sahu et al. [11] investigated and compared different portfolio optimization methods and short-term investment strategies for the cryptocurrency market. Focusing on the top ten cryptocurrencies by market capitalization, the study employs high-frequency data to evaluate the effectiveness of two primary optimization strategies – Sharpe ratio maximization and kurtosis minimization – with the aim to determine the strategy that could offer the best performance in terms of optimizing returns and managing risks, particularly for short-term investment horizons. Chen [12] evaluated the applicability and effectiveness of Modern Portfolio Theory (MPT) by examining whether traditional MPT principles, such as diversification and risk-return optimization, remain relevant and effective when applied to portfolios that incorporate cryptocurrencies. Jelescovic et al. [13] assessed the effectiveness of integrating cryptocurrencies into traditional portfolios using a GARCH-Copula model within the Markowitz framework to determine if these techniques improve portfolio optimization, particularly by enhancing the Sharpe ratio and overall stability. The study examined the risk-return profile across three portfolio types: traditional assets, cryptocurrencies, and a combination of both (hybrid).

Kim et al. [14] empirically investigated the impact of incorporating cryptocurrencies into global asset portfolios using ensemble approaches and tracing strategies. The study specifically evaluates the performance of portfolios with cryptocurrency allocations of 1%, 3%, and 5% across several portfolio optimization strategies, including minimum variance, maximum diversification, equal risk contribution, and hierarchical risk parity. The study demonstrated how different allocation ratios of cryptocurrencies affect portfolio performance metrics such as returns, volatility, Sharpe ratio, and maximum drawdown. Maghsoodi [15] developed a hybrid decision support system for cryptocurrency portfolio management, combining time series forecasting using the Prophet Forecasting Model (PFM) with an enhanced CLUS-MCDA II algorithm. The study aims to guide investors in optimizing portfolio allocation with over 70 cryptocurrencies by integrating advanced clustering methods like DBSCAN with multi-criteria decision analysis (MCDM) techniques such as VIKOR and MULTIMOORA.

2.2. Research gap and hypotheses development

Despite the growing interest in cryptocurrency investments, the literature on portfolio allocation under uncertainty in the cryptocurrency market remains limited. The unique characteristics of this market, including extreme volatility and regulatory uncertainties, necessitate the development of specialized portfolio management and risk mitigation strategies. These challenges highlight the critical need for integrating uncertainty modeling techniques into optimization frameworks. Although traditional risk measures and their applications in portfolio optimization have been widely studied, the use of fuzzy logic to address the inherent uncertainty and ambiguity in cryptocurrency markets is still underexplored. Moreover, essential practical constraints such as cardinality and floor and ceiling constraints are frequently overlooked in existing models, resulting in a significant gap in the creation of robust and practical portfolio strategies. This study seeks to bridge these gaps by applying the Credibilistic CVaR criterion to cryptocurrency portfolio allocation, incorporating practical constraints to enhance the realism and applicability of the proposed models.

Drawing from the insights gained through the reviewed literature, we propose the following hypotheses to address the identified research gaps and advance the understanding of portfolio optimization in cryptocurrency markets:

- The Credibilistic CVaR framework with trapezoidal fuzzy variables will optimize cryptocurrency portfolios more effectively than traditional models.
- Practical constraints like cardinality, and floor and ceiling constraints will create well-diversified portfolios with improved risk-adjusted returns.
- The proposed model will enhance risk management and decision-making for cryptocurrency investors.

3. PRELIMINARIES

3.1. Fuzzy set theory

The classical set theory evaluates elements according to binary criteria, determining whether an element is either a member of a set or not. In contrast, fuzzy set theory presents mathematical frameworks that permit a more nuanced evaluation of element membership within a set. This is represented through a membership function that assigns each element a membership grade between zero and one. This approach effectively addresses the vagueness and uncertainty inherent in various scenarios, which can be defined as follows:

3.1.1 Fuzzy set

Let X represent a universe, with its generic element denoted as x. A fuzzy set A can be characterized as a collection of ordered pairs defined within the universe X, expressed as follows:

$$A = \{(x, \mu_A(x) |\ x \in X)\} \quad (1)$$

where, $\mu_A(x)$ represents the membership function or the degree of membership of an element $x \in X$, which is defined within the real interval [0,1]. The value of $\mu_A(x)$ indicates how strongly $x \in X$ belongs to the set A. As previously noted, it is typical for experts to convey their assessments using fuzzy numbers in practical scenarios.

3.1.2 Trapezoid fuzzy number

Let $\tilde{A}$ be represented as a trapezoid fuzzy number defined by the parameters (a_1, a_2, a_3, a_3), where a_1, a_2, a_3 and a_4 are real numbers with $a_1 \leq a_2 \leq a_3 \leq a_4$. The membership function $\mu_A(x)$ corresponding to $\tilde{A}$ is defined as:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \in (-\infty, a_1) \\ \dfrac{x - a_1}{a_2 - a_1}, & x \in [a_1, a_2] \\ 1, & x \in [a_2, a_3] \\ \dfrac{a_4 - x}{a_4 - a_3}, & x \in [a_3, a_4] \\ 0, & x \in (a_4, +\infty) \end{cases} \quad (2)$$

The membership function for a trapezoidal fuzzy number forms a trapezoid shape, where the fuzzy number $\tilde{A}$ has a constant maximum membership value of 1 over the interval $[a_2, a_3]$. The membership values decrease linearly towards 0 outside this interval, on either side towards a_1 and a_4.

3.2. Credibility theory

Credibility theory, initially introduced by Liu [8] and extended in Liu's subsequent work [9], is a robust mathematical framework designed for the analysis and modeling of fuzzy phenomena. This theory plays a crucial role in the development of credibility fuzzy programming, a method that enables decision-makers to quantify and assess their level of confidence in achieving specific constraints under uncertainty. Credibility theory offers a flexible approach to handling various types of fuzzy data, such as triangular and trapezoidal fuzzy numbers, which are commonly used to represent uncertain information in decision-making processes. By incorporating these fuzzy numbers, the theory allows for a more nuanced and realistic representation of uncertainty, which is often encountered in real-world scenarios. Liu [8], [9] presents the fundamental definitions and notations that underpin credibility theory. These include the concept of the credibility measure, a key tool used to evaluate the likelihood or degree of confidence that a certain event or condition will occur within a fuzzy environment. As noted by Giunta et al. [16], the calculation of the credibility measure is conducted as follows:

$$Cr\{\xi \in A\} = \frac{1}{2}(Pos\{\xi \in A\} + Nes\{\xi \in A\}) \quad (3)$$

where, $Pos\{\xi \in A\}$ represents the possibility measure of the event $\{\xi \in A\}$, while $Nes\{\xi \in A\}$ denotes the necessity measure of the same event. These two measures are fundamental to fuzzy set theory:

- Possibility Measure (Pos): This measure quantifies the maximum degree of membership within the fuzzy set A, essentially reflecting the most plausible degree to which the event $\{\xi \in A\}$ can occur. It is defined as:

$$Pos\{\xi \in A\} = \sup \mu(x)_{x \in A} \quad (4)$$

- Necessity Measure (Nes): The necessity measure quantifies the degree to which the event $\{\xi \in A\}$ is certain, calculated as the complement of the maximum degree of membership in the complement set A^c. It is expressed as:

$$Nes\{\xi \in A\} = 1 - \sup \mu(x)_{x \in A^c} \quad (5)$$

Given that $Pos\{\xi \in A\} = \sup \mu(x)_{x \in A}$ and $Nes\{\xi \in A\} = 1 - \sup \mu(x)_{x \in A^c}$, the credibility measure can also be expressed as:

$$Cr\{\xi \in A\} = \frac{1}{2}(\sup \mu(x)_{x \in A} + 1 - \sup \mu(x)_{x \in A^c}) \quad (6)$$

This formulation highlights that the credibility measure balances both the possibility and the necessity of the event $\{\xi \in A\}$, providing a comprehensive assessment of its likelihood in a fuzzy environment. When considering a specific fuzzy event characterized by $\{\xi \leq r\}$, where r is a real number, the credibility measure is given by:

$$Cr\{\xi \leq r\} = \frac{1}{2}(\sup \mu(x)_{x \leq r} + 1 - \sup \mu(x)_{x > r}) \quad (7)$$

This equation provides a way to evaluate the credibility that a fuzzy variable ξ will take a value less than or equal to a given real number r. The measure takes into account both the maximum membership value within the interval $(-\infty, r]$ and the complement membership value in the interval $(r, +\infty)$, thus offering a balanced view of the likelihood of the event. The expected value of a fuzzy variable ξ is another important concept within credibility theory, reflecting the "average" outcome of ξ when considering its fuzzy nature. It is calculated using the following expression:

$$E[\xi] = \int_0^{+\infty} Cr\{\xi \geq r\}dr - \int_{-\infty}^{0} Cr\{\xi \leq r\}dr \quad (8)$$

This formula integrates the credibility measures over the entire real line, effectively capturing the expected value of ξ by balancing the contributions from both the positive and negative domains. The first integral considers the credibility of ξ being greater than or equal to r, while the second integral accounts for the credibility of ξ being less than or equal to r. The subsequent sections of the paper focus on the application of the credibility measure to specific types of fuzzy variables, particularly triangular and trapezoidal fuzzy numbers.

4. THE PROPOSED PORTFOLIO OPTIMIZATION MODEL

In this section, we introduce a comprehensive portfolio optimization model that integrates the CVaR measure within the framework of credibility theory.

4.1 Conditional Value at Risk (CVaR)

A primary goal of risk management is to evaluate and improve the performance of financial investments by carefully considering the risks taken to generate profits. One widely adopted method for quantifying these risks is VaR, which has become a standard tool for measuring potential losses. However, despite its popularity, VaR has several inherent limitations, particularly its inability to capture extreme or tail-end risks fully. In response to these shortcomings, CVaR has gained recognition as a more advanced and comprehensive risk measure. Often referred to as mean excess loss, mean shortfall, or tail VaR, CVaR provides a deeper understanding of risk by focusing on the potential losses that exceed the VaR threshold.

CVaR is a prominent risk measure that extends beyond traditional VaR by focusing on the tail of the loss distribution. Unlike VaR, which only provides the maximum loss within a specified confidence level, CVaR captures the expected loss that occurs beyond this VaR threshold, thereby offering a more comprehensive view of potential extreme losses. This characteristic makes CVaR especially useful in scenarios where tail risk is a significant concern, such as in financial risk management, insurance, and portfolio optimization. One of the key advantages of CVaR is its coherence, meaning it satisfies desirable mathematical properties like sub-additivity, translation invariance, monotonicity, and positive homogeneity. Sub-additivity ensures that the risk of a combined portfolio is never greater than the sum of the risks of individual portfolios, promoting diversification. These properties make CVaR a robust and reliable measure for assessing risk in various contexts. The CVaR of a random variable ξ at a confidence level α is mathematically expressed as:

$$CVaR(x, \eta) = \eta + (1-\alpha)^{-1} \int_{\xi \varepsilon R^n} [f(X, \xi) - \eta]^+ p(\xi) d\xi \quad (9)$$

Where $[f(X,\xi)-\eta]^+$ is expressed as:

$$[f(X,\xi)-\eta]^+ \stackrel{\text{def}}{=} \begin{cases} f(X,\xi)-\eta & \text{if} \quad f(X,\xi)-\eta>0 \\ 0 & \text{if} \quad f(X,\xi)-\eta\leq 0 \end{cases} \tag{10}$$

Here, η represents the VaR threshold, α is the confidence level, $f(X,\xi)$ denotes the loss function, and $p(\xi)$ is the probability density function of ξ. This formulation leverages linear programming techniques to efficiently calculate CVaR, making it highly applicable in financial optimization scenarios.

4.2. CVaR under credibility theory

Incorporating credibility theory into the calculation of CVaR allows for the treatment of uncertainty in a more flexible and nuanced manner. Credibility theory, particularly useful in situations with fuzzy data, provides an alternative approach to traditional probability-based methods. For a fuzzy variable ξ and a confidence level $\alpha \in (0,1]$, the VaR under credibility theory can be defined as:

$$\xi_{\mathrm{VaR}}(\alpha) = -\sup\{x \mid \mathrm{Cr}\{\xi \leq x\} \leq \alpha\} \tag{11}$$

This definition identifies the maximum value of x for which the credibility measure $\mathrm{Cr}\{\xi \leq x\}$ is less than or equal to α, providing a fuzzy analog to the traditional VaR measure. Similarly, the alternative expression for VaR under credibility theory is:

$$\xi_{\mathrm{VaR}}(\alpha) = -\inf\{x|\mathrm{Cr}\{\xi \leq x\} \geq \alpha\} = -\inf\{x|\Phi(x) \geq \alpha\} = -\Phi^{-1}(\alpha) \tag{12}$$

Where $\Phi(x)$ represents the cumulative credibility distribution function.

The CVaR under credibility theory, denoted as ξ_{CVaR}, is obtained by integrating the VaR function over the confidence interval:

$$\xi_{\mathrm{CVaR}} = \frac{1}{1-\alpha}\int_{\alpha}^{1} \xi_{\mathrm{VaR}}(r)dr \tag{13}$$

Considering the information presented, the CVaR for a triangular fuzzy variable characterized by parameters $\xi = (a_1, a_2, a_3)$ and a confidence level $\alpha \in (0,1]$ can be calculated in the following formula:

$$\xi_{\mathrm{CVaR}}(\alpha) = \begin{cases} \alpha a_1 - (1+\alpha)a_2 & \alpha \in (0, 0.5] \\ (\alpha-1)a_2 - \alpha a_3 & \alpha \in (0.5, 1] \end{cases} \tag{14}$$

Similarly, the CVaR for a trapezoidal fuzzy variable characterized by parameters $\xi = (a_1, a_2, a_3, a_4)$ and a confidence level $\alpha \in (0,1]$ can be calculated as follows:

$$\xi_{\mathrm{CVaR}}(\alpha) = \begin{cases} \alpha a_1 - (1+\alpha)a_2 & \alpha \in (0, 0.5] \\ (\alpha-1)a_3 - \alpha a_4 & \alpha \in (0.5, 1] \end{cases} \tag{15}$$

4.3 Additional practical constraints

In real-world portfolio optimization, several practical constraints must be considered to ensure that the model reflects actual investment scenarios. Incorporating these constraints enhances the realism of the portfolio selection process and aligns the model with practical investment considerations.

4.3.1 Cardinality constraint

The cardinality constraint limits the number of assets that can be included in a portfolio. This constraint is crucial for managing transaction costs and ensuring portfolio diversification. The selection status of an asset is represented by the binary variable Z_i, and the constraint is formulated as follows:

$$\sum_{i=1}^{N} Z_i = K \tag{16}$$

and

$$Z_i \in \{0,1\}, i = 1,2,\ldots,N \tag{17}$$

Where N is the total number of available assets, and K is the maximum number of assets allowed in the portfolio.

4.3.2 Floor and ceiling constraints

Floor and ceiling constraints, also known as buy-in thresholds, establish the minimum and maximum limits for the proportion of the portfolio allocated to each asset. These constraints ensure that the portfolio does not become overly concentrated in a single asset or include insignificant positions. The constraints are expressed as:

$$l_i Z_i \le x_i \le u_i Z_i, \quad i = 1,2,\ldots,N \tag{18}$$

and

$$0 \le l_i \le u_i \le 1 \tag{19}$$

Where, l_i and u_i represent the lower and upper bounds on the proportion x_i allocated to asset i.

4.4 Proposed portfolio optimization problem formulation

This section introduces the proposed Credibilistic CVaR model, tailored for portfolio optimization within the framework of trapezoidal fuzzy variables. The model is designed to handle uncertainty in financial markets by incorporating fuzzy variables, which better reflect the imprecise and ambiguous nature of real-world data compared to traditional probabilistic methods. By employing trapezoidal fuzzy numbers, the model offers a more flexible and realistic approach to assessing risk and making investment decisions. The Credibilistic CVaR model is particularly suited for scenarios where the confidence level, denoted by α, plays a critical role in determining the degree of risk aversion. In this model, the confidence level is restricted to the interval $\alpha \in (0, 0.5]$, ensuring that the assessment of risk remains conservative.

This constraint on α allows the model to capture extreme downside risk more effectively, making it a valuable tool for investors who prioritize avoiding significant losses. In addition to the focus on CVaR minimization, the proposed model also incorporates cardinality, floor, and ceiling constraints. These constraints add further realism and practicality to the portfolio optimization process. The cardinality constraint limits the number of assets in the portfolio, reflecting real-world limitations such as transaction costs and regulatory requirements. Meanwhile, the floor and ceiling constraints set lower and upper bounds on the allocation to each asset, ensuring a diversified and balanced portfolio. Together, these constraints enhance the model's ability to generate feasible and well-structured portfolios that align with investors' practical concerns. The proposed portfolio optimization model is set up as follows:

$$\text{Min CVaR} = \sum_{i=1}^{n} x_i[\alpha a_{1i} - (1+\alpha)a_{2i}] \tag{20}$$

S. t.

$$\sum_{i=1}^{n} x_i \frac{a_{1i} + a_{2i} + a_{3i} + a_{4i}}{4} \geq R \tag{21}$$

$$l_i Z_i \leq x_i \leq u_i Z_i \tag{22}$$

$$\sum_{i=1}^{n} Z_i = K \tag{23}$$

$$\sum_{i=1}^{n} x_i = 1 \tag{24}$$

$$Z_i = \{0,1\} \tag{25}$$

$$x_i \geq 0, \quad i = 1, 2, \dots, n \tag{26}$$

Where, Equation (20) defines the objective of the model, which is to minimize the Credibilistic CVaR. Equation (21) introduces the expected return constraint, ensuring that the portfolio achieves at least the required return R, balancing the trade-off between risk and return. Equation (22) outlines the floor and ceiling constraints, which enforce lower and upper bounds on each asset's allocation, where Z_i is a binary variable indicating whether an asset is included in the portfolio. Equation (23) describes the cardinality constraint, limiting the number of assets in the portfolio to K, which reflects practical considerations like transaction costs and portfolio manageability. Equation (24) sets forth the budget constraint, ensuring that the entire budget is allocated across the selected assets. Lastly, Equations (25) and (26) establish the binary and non-negativity constraints, mandating that Z_i is binary (indicating the inclusion or exclusion of an asset) and that x_i is non-negative, thereby preventing short positions.

5. NUMERICAL EXPERIMENTS AND SENSITIVITY ANALYSIS

In this section, we implement the proposed fuzzy portfolio optimization model using a real-world case study from the cryptocurrency market. The aim is to demonstrate the practical applicability of the model in constructing an optimal portfolio that balances risk and returns in the highly volatile and uncertain environment of digital assets. To achieve this, we selected a diverse dataset of 47 cryptocurrencies from https://www.investing.com/crypto website, representing a broad spectrum of the market.

Table 1 presents a trapezoidal fuzzy data set for the returns of various cryptocurrencies, capturing the uncertainty and variability in their historical performance. Each asset is represented by four key points: the minimum return, a lower median, an upper median, and the maximum return. These points form a trapezoidal fuzzy number that reflects the range of possible outcomes for each cryptocurrency, accounting for both optimistic and pessimistic scenarios. This fuzzy representation is particularly useful in the context of cryptocurrency markets, where uncertainty and rapid price fluctuations are common. By using trapezoidal fuzzy numbers, this approach provides a more flexible and realistic model for assessing the risk and return profiles of these digital assets.

Table 1. Trapezoid fuzzy data set for cryptocurrencies return.

ID	Crypto	Trapezoidal Fuzzy Data	ID	Crypto	Trapezoidal Fuzzy Data
A_1	Ethereum	(-0.101, -0.028, 0.118, 0.19)	A_{25}	Gnosis	(-0.181, -0.093, 0.082, 0.17)
A_2	Solana	(-0.085, -0.023, 0.103, 0.165)	A_{26}	Aave	(-0.216, -0.102, 0.125, 0.239)
A_3	Tron	(-0.082, -0.031, 0.071, 0.123)	A_{27}	Arweave	(-0.189, -0.016, 0.33, 0.503)
A_4	Binance	(-0.136, -0.066, 0.074, 0.144)	A_{28}	dYdX	(-0.169, -0.08, 0.099, 0.189)
A_5	Bitcoin	(-0.209, 0.065, 0.614, 0.889)	A_{29}	Near	(-0.182, -0.015, 0.32, 0.488)
A_6	Chainlink	(-0.166, -0.026, 0.253, 0.393)	A_{30}	1inch	(-0.204, -0.109, 0.082, 0.178)
A_7	Arbitrum	(-0.17, -0.089, 0.073, 0.154)	A_{31}	Oasis	(-0.366, -0.215, 0.087, 0.238)
A_8	Sui	(-0.141, 0.029, 0.368, 0.538)	A_{32}	Sei	(-0.165, -0.025, 0.254, 0.394)
A_9	Avalanche	(-0.148, 0.056, 0.465, 0.67)	A_{33}	Harmony	(-0.195, -0.1, 0.091, 0.186)
A_{10}	Polygon	(-0.145, -0.05, 0.141, 0.237)	A_{34}	Decentralan	(-0.204, -0.084, 0.154, 0.273)
A_{11}	Curve	(-0.159, -0.062, 0.131, 0.228)	A_{35}	Kava	(-0.182, -0.069, 0.158, 0.271)
A_{12}	Aptos	(-0.159, -0.025, 0.242, 0.376)	A_{36}	Ronin	(-0.197, -0.095, 0.109, 0.21)
A_{13}	Optimism	(-0.171, -0.067, 0.14, 0.244)	A_{37}	Vechain	(-0.173, -0.027, 0.265, 0.411)
A_{14}	CORE	(-0.176, -0.07, 0.142, 0.248)	A_{38}	Litecoin	(-0.291, -0.001, 0.579, 0.869)
A_{15}	Uniswap	(-0.171, -0.036, 0.233, 0.368)	A_{39}	EOS	(-0.194, -0.112, 0.052, 0.134)
A_{16}	Mantle	(-0.144, -0.012, 0.252, 0.384)	A_{40}	Celo	(-0.188, -0.045, 0.241, 0.384)
A_{17}	Cronos	(-0.15, -0.055, 0.135, 0.229)	A_{41}	Fantom	(-0.203, -0.024, 0.333, 0.512)
A_{18}	Ondo	(-0.123, -0.04, 0.127, 0.21)	A_{42}	MultiversX	(-0.15, -0.032, 0.204, 0.322)
A_{19}	Numbers	(-0.198, -0.084, 0.144, 0.258)	A_{43}	Stacks	(-0.182, -0.058, 0.191, 0.315)
A_{20}	Pancakeswa	(-0.184, -0.091, 0.094, 0.186)	A_{44}	Monero	(-0.101, 0.007, 0.222, 0.33)
A_{21}	Maker	(-0.189, -0.068, 0.174, 0.294)	A_{45}	Cosmos	(-0.235, -0.115, 0.125, 0.245)
A_{22}	Thorchain	(-0.166, -0.032, 0.237, 0.372)	A_{46}	Tezos	(-0.228, 0.116, 0.805, 1.15)
A_{23}	TON	(-0.166, -0.026, 0.253, 0.393)	A_{47}	Algorand	(-0.22, -0.087, 0.18, 0.314)
A_{24}	Cardano	(-0.172, -0.059, 0.169, 0.282)			

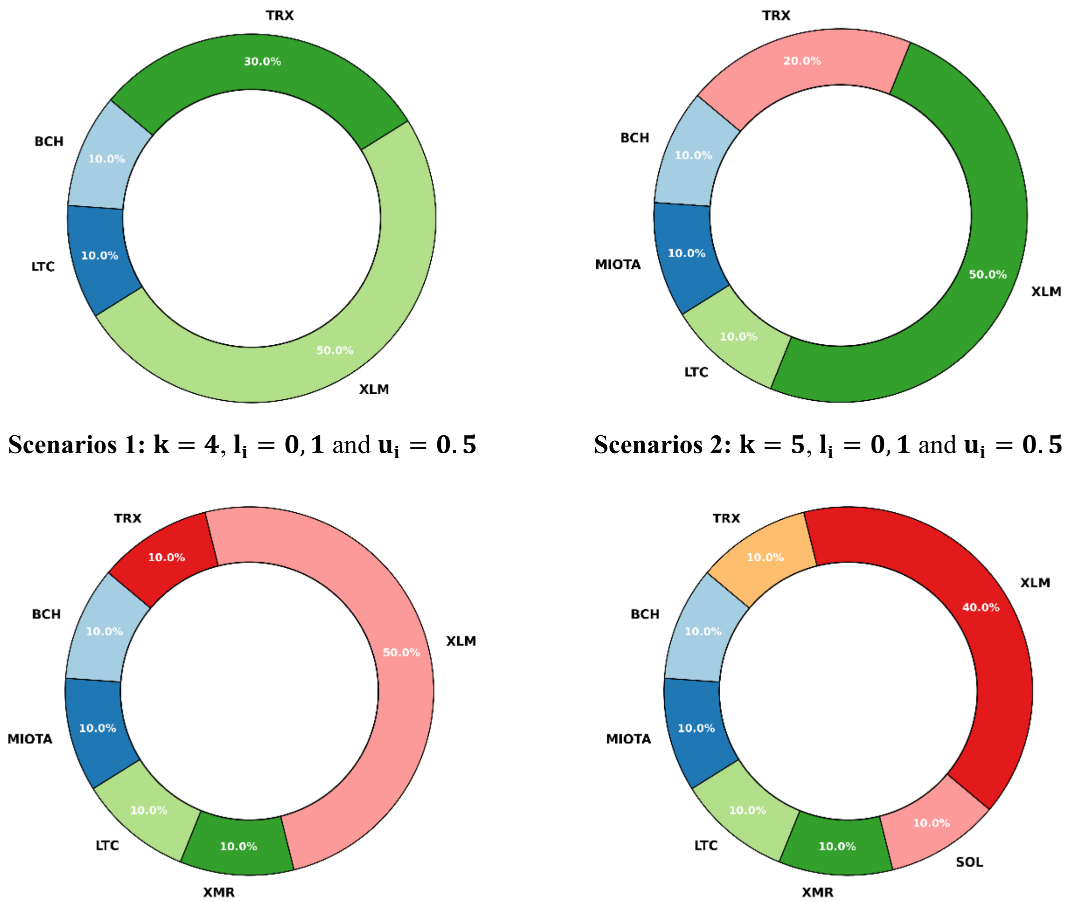
The results of the proposed model, evaluated under various scenarios, are summarized in Table 2. These scenarios were designed to cater to different types of investors by adjusting constraints on the number of assets and the bounds for investment allocations. In the formulation of the proposed model, the α value was set to 0.05. This parameter is used in the optimization of the Conditional CVaR objective function, as described by Equation (20). The analysis included scenarios with different values for k (the number of assets in the portfolio) and various lower (l_i) and upper (u_i) bounds on asset allocations to evaluate the model's robustness and adaptability under various conditions.

Table 2. The results of proposed model under different scenarios.

k	l_i	u_i	Objective Function	x_i							
				x_4	x_{18}	x_{23}	x_{26}	x_{28}	x_{30}	x_{32}	x_{35}
k = 4	0.1	0.5	1.088	0.1	-	0.1	-	-	-	0.5	0.3
	0.1	0.3	0.845	0.1	-	0.3	-	-	-	0.3	0.3
k = 5	0.1	0.5	0.978	0.1	0.1	0.1	-	-	-	0.5	0.2
	0.1	0.3	0.842	0.1	0.1	0.2	-	-	-	0.3	0.3
k = 6	0.1	0.5	0.866	0.1	0.1	0.1	0.1	-	-	0.5	0.1
	0.1	0.3	0.836	0.1	0.1	0.1	0.1	-	-	0.3	0.3
k = 7	0.1	0.5	0.737	0.1	0.1	0.1	0.1	-	0.1	0.4	0.1
	0.1	0.3	0.722	0.1	0.1	0.1	0.1	-	0.1	0.3	0.2

Figure 1 presents the portfolio allocations under the different scenarios through pie charts, providing a visual representation of the asset distributions for each case. These charts illustrate how the proposed model adjusts asset allocations according to varying constraints on the number of assets and investment bounds, as detailed in Table 2.

As evidenced in Table 2 and Figure 1 A_{29} and Tron A_{33} consistently receive the largest allocations across all scenarios, underscoring their significance in the optimized portfolios. This outcome is not merely coincidental but is deeply rooted in the underlying financial characteristics of these assets.



Scenarios 1: $\mathbf{k = 4, l_i = 0,1}$ and $\mathbf{u_i = 0.5}$ **Scenarios 2:** $\mathbf{k = 5, l_i = 0,1}$ and $\mathbf{u_i = 0.5}$

Scenarios 3: $k = 6$, $l_i = 0{,}1$ and $u_i = 0.5$ **Scenarios 4**: $k = 7$, $l_i = 0{,}1$ and $u_i = 0.5$

Fig. 1. The presentation of portfolios under different scenarios.

7. CONCLUSIONS

This study presents a new method for cryptocurrency portfolio optimization within a credibilistic CVaR framework. It applies credibility theory and trapezoidal fuzzy variables to capture the deep uncertainty and volatility of digital assets. Compared with standard probabilistic approaches, the model gives a more adaptive way to manage risk. Practical constraints make the model usable in real markets, reflecting transaction costs, regulations and the need for diversified allocation. Empirical tests show the model builds well-diversified portfolios that balance risk and return, offering a useful tool for investors in a fast-moving crypto market. The paper also points to clear paths for future work. Researchers could test alternative fuzzy numbers like triangular or Gaussian forms. Machine learning, deep learning, and AI could improve fuzzy parameter estimation and serve as asset preselection tools, refining the optimization. Extending the model to multi-period horizons would let it track price dynamics and support longer-term strategies. The authors stress that cryptocurrencies remain volatile and largely driven by speculation and sentiment, influenced by use cases, regulation, adoption and news. The asset sample used was illustrative, not an investment recommendation. The method is flexible and results will vary with market conditions. Treat the findings as a methodological contribution and perform due diligence before investing.

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