Estimating Shear Wave Velocity Structure through Multichannel Analysis of Surface Waves in Geotechnical Site Investigations

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ABSTRACT

The shear resistance coefficient is a crucial parameter required in geotechnical studies of engineering project sites. Shear wave velocity is also utilized to determine soil stiffness, which represents one of the fundamental properties of the ground in engineering science. Given that obtaining such data through non-geophysical methods entails substantial costs, seismic methods represent the most suitable approach for this purpose. These non-destructive seismic techniques provide designers with the aforementioned data.

Conventional seismic methods, such as the Refraction Microtremor (ReMi) or similar boundary-breaking techniques used to determine shear wave velocity, exhibit certain limitations. In this study, while introducing the Multichannel Analysis of Surface Waves (MASW) method and comparing it with conventional methods, shear wave velocities have been calculated in several study areas. This method employs the inversion of Rayleigh wave dispersion curves derived from multichannel seismic data.

To evaluate the efficacy of the proposed method, the developed software was applied to two sets of real seismic data from different projects. A comparison of results obtained from this method with those from refraction seismic indicates that, in addition to consistency in shear wave velocity values, the MASW method demonstrates fewer limitations and more advantages.

1. **Keywords:** Shear wave velocity, Multichannel Analysis of Surface Waves (MASW), Rayleigh Wave, Dispersion Curve Inversion

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2. INTRODUCTION

The critical importance of shear wave velocity (Vs) distributions across structural engineering, geotechnical investigations, and geophysical projects has driven demand for rapid, non-destructive methodologies that overcome the inherent energy limitations of conventional S-wave sources. This study presents an innovative approach leveraging Rayleigh wave propagation—a class of elastic surface waves—to address this fundamental challenge.

Building upon foundational work by the Kansas Geological Survey [1], we utilize the fundamental mode of Rayleigh **waves** in multichannel seismic records to derive high-resolution Vs profiles for near-surface characterization. Our methodology employs a 24-channel acquisition system to capture wavefield properties with optimal signal fidelity.

This research advances the field through dual dispersion curve extraction techniques:

1. Cross-correlation spectral analysis determining phase velocities via Fourier-domain trace processing
2. Stretch function transformation generating Swept Frequency Records (SFR) for dispersion curve derivation

The extracted dispersion curves undergo rigorous non-linear inversion to resolve Vs distributions. Crucially, both methods overcome limitations of refraction seismology by accurately characterizing velocity inversion layers—where standard techniques fail—due to the sensitivity of Rayleigh wave dispersion to velocity reversals [2].

3. Research Methodology

The foundation of the Multichannel Analysis of Surface Waves (MASW) methodology relies on generating Rayleigh-type surface waves and recording them through an array of receivers along a designated profile. This approach leverages the dispersion characteristics of Rayleigh waves to determine near-surface shear wave velocity distribution, requiring precise computation of the fundamental mode phase velocity [3].

Central to this technique is the spectral analysis of Rayleigh waves in the Fourier domain, where systematic processing extracts phase velocities of frequency components within the fundamental mode. The method capitalizes on the inherent frequency-dependent propagation behavior of surface waves, transforming spectral signatures into quantifiable geotechnical parameters [4].

Like all seismic techniques, MASW implementation follows three critical operational phases:

1. Data Acquisition: Strategic deployment of energy sources and geophone arrays to capture high-fidelity Rayleigh wave signatures;
2. Signal Processing: Advanced transformation of raw seismic records into dispersion curves using Fourier analysis and inversion algorithms;
3. Geophysical Interpretation: Conversion of dispersion data into vertically stratified shear wave velocity models through constrained inversion procedures.

4. Data

3.1 Data Processing

The processing of acquired seismic data for near-surface geophysical characterization comprises three principal stages: identification of Rayleigh waves on the seismic record, extraction of the Rayleigh wave dispersion curve, and inversion computations to determine the shear wave velocity (V_S) profile. This workflow begins with identifying Rayleigh waves, which dominate later seismic arrivals through their distinctive elliptical retrograde particle motion, low-frequency content (typically 1–30 Hz), exponential amplitude decay with depth, and characteristic dispersion patterns appearing as hyperbolic trends in shot gathers. These surface waves are isolated from body waves and noise using advanced signal processing techniques such as frequency-wavenumber (f-k) analysis, tau-p transforms, or time-offset filtering. Following identification, the dispersion curve—quantifying the relationship between Rayleigh wave phase/group velocity and frequency is extracted through methods like Multichannel Analysis of Surface Waves (MASW) that transforms shot gathers into frequency-velocity domains, Spectral Analysis of Surface Waves (SASW) based on cross-correlation of receiver pairs, or wavefield transformation techniques. This curve serves as the primary input for the subsequent inversion stage, where nonlinear optimization algorithms iteratively minimize misfit between observed data and theoretical dispersion curves simulated for layered earth models. The inversion incorporates regularization constraints to address solution non-uniqueness and employs global search methods (e.g., Monte Carlo, genetic algorithms) or gradient-based approaches (e.g., Levenberg-Marquardt) to derive a depth-dependent (V_S) profile. Throughout this process, key considerations include the interdependence of stages, where errors in initial wave identification propagate to later phases; resolution limitations dictated by receiver geometry and signal-to-noise ratio; and validation through uncertainty quantification or borehole data correlation. The resulting (V_S) profile enables critical geotechnical assessments of soil stiffness, liquefaction potential, bedrock depth, and seismic site response, with commercial (e.g., SeisImager) or open-source tools (e.g., Geopsy) facilitating the end-to-end transformation of raw seismic waveforms into subsurface engineering models.

3.2 Data Interpretation

The extraction of the Rayleigh wave dispersion curve from seismic records is a critical phase in surface wave analysis. This study implements two specific methods: (1) Multichannel Analysis of Surface Waves

(MASW) using phase-shift transformation of shot gathers to generate frequency-velocity spectra, and (2) Spectral Analysis of Surface Waves (SASW) deriving phase velocity from cross-correlation of receiver pairs[5]. Dispersion curves (phase velocity vs. frequency) are then picked from peak energy trends in the transformed data. These curves serve as inputs for nonlinear inversion, where misfit between observed and theoretical dispersion values is minimized via optimization algorithms (e.g., Levenberg-Marquardt) to derive a layered shear wave velocity (V_S) profile. Finally, the resolved V_S model undergoes validation through uncertainty quantification and comparison with borehole data. The steps of data preparation and processing are shown in the diagram in Figure 1.

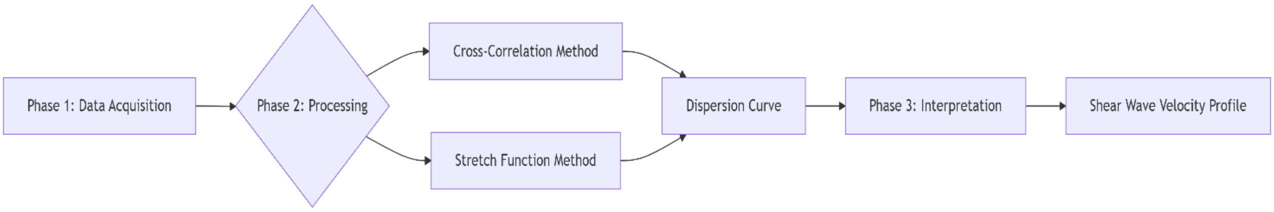

Fig. 1. schematically outlines the data processing workflow[6].

5. Spectral Analysis Method

The transformation of data from the time domain to the frequency domain relies fundamentally on the Fourier transform. If the function U(x,t) represents the seismic record in the time-space domain, the Fourier transform of the data along the time axis can be expressed as follows:

$$U\,(x, \omega) \;= U\,(x, t)e^{i\omega t} \tag{1}$$

Various processing operations can be performed on a signal that has been transformed into the frequency domain. The spectrum $S_x(f)$ of a signal is, in fact, the Fourier transform of the time-domain recorded signal. This spectrum is employed to identify the dominant frequency and its corresponding absolute amplitude and phase within a seismic record [7].

The autocorrelation spectrum $G_{xx}(f)$, is equal to the product of the spectrum $S_x(f)$ and its complex conjugate. The magnitude of the autocorrelation spectrum is equal to the square of the magnitude of the linear spectrum $S_x(f)$, and is expressed as follows:

$$G_{xx}(f) = S_y^*(f).S_x(f) \tag{2}$$

The amplitude spectrum of the cross-correlation function indicates the common frequency components present in both records. The phase information from the function $G_{xy}(f)$ can be employed to determine the phase difference between the two records. The cross-correlation function spectrum serves as an effective tool for determining the relative phase difference between two signals resulting from a time delay. Consequently, the cross-correlation function spectrum can be utilized to determine the dispersion curve. Since a 360-degree phase shift corresponds to one period (T), the surface wave travel time for different frequencies can be determined from the following relation:

$$\Delta t = T \times \frac{\Delta\varphi}{360} \tag{3}$$

where Δt is the travel time difference of a specific frequency component between two receivers at a known offset, Δϕ is the phase shift, and T is the period of the target frequency [8]. Given the known inter-receiver offset, the phase velocity (V_ϕ) and wavelength (λ) of the target frequency can be calculated as:

$$V_\varphi = \frac{\Delta x}{\Delta t}\ , \qquad \lambda = V_\varphi\ ,\ T = \frac{V_\varphi}{f} \tag{4}$$

By repeating this procedure for all frequency components and for all receiver pair offsets, the requisite data for constructing the dispersion curve are obtained. Given that the frequency content of Rayleigh waves changes with distance from the source, it is expected that determining the cross-correlation phase spectrum between receivers at multiple offsets will yield discrete segments of the dispersion curve. These segments typically exhibit significant overlap, enabling the derivation of a continuous dispersion curve through their integration [4].

6. SFR Method

The second method employed in this study to determine the dispersion curve from a common-source seismic record containing Rayleigh waves is the SFR method (Swept-Frequency Record). Three key parameters define this record format:

f_1: Lowest frequency present in the Rayleigh waves
f_2: Highest frequency present in the Rayleigh waves
T: Sweep length (duration)

Typically, using either the lowest frequency (f_1) present in the Rayleigh waves or their longest wavelength (λ_{max}), the maximum investigation depth (Z_{max}) can be estimated as follows [9]:

$$Z_{max} = \frac{\lambda_{max}}{2} = \frac{C_1}{2f} \tag{5}$$

where C_1 is the phase velocity at frequency f_1.

The lowest recorded frequency is typically controlled by the natural frequency of the geophones and the position of the energy source. The highest frequency value is determined through evaluation of the seismic record's frequency spectrum. The sweep duration (T) must be sufficiently long to reliably analyze the frequency variations of the Rayleigh waves. However, if the frequencies are appropriately selected, a record length between 5 to 10 seconds will generally be adequate [10].

Let r(t) represent a seismic trace generated by a compressional wave (P-wave) energy source and recorded by a vertical-component receiver. The SFR form of this trace can be obtained by convolving it with a stretch function s(t):

$$SFR(t) = r(t) * s(t) \tag{6}$$

The stretch function is a mathematical operator that generates the frequency variation over time to realize the frequency sweep. The following function represents a suitable choice for s(t) [10]:

$$S(t) = \sin\left[\pi . T . f_1 \frac{\mathrm{Ln}\left[1 + \frac{t}{T} . \left(\frac{f_2}{f_1} - 1\right)\right]}{\frac{f_2}{f_1} - 1}\right] \tag{7}$$

where f_1, f_2, and T denote the lowest frequency, highest frequency, and record length (or sweep duration) of s(t), respectively.

7. Dispersion Curve Inversion

The inversion of Rayleigh wave dispersion curves constitutes an iterative optimization procedure formally termed Iterative Forward Modeling. This nonlinear process commences with the selection of an initial 1D subsurface model characterizing the shear-wave velocity (V_S) structure beneath the receiver array. Initial model parameterization defines:

- Layer thicknesses (hi)
- V_S values
- V_P/V_S ratios
- Densities (ρ_i)

for N subsurface layers. Model initialization leverages geophysical constraints:

- S-wave refraction data directly provides V_S estimates
- First-arrival P-wave times convert to V_S

The theoretical dispersion curve is computed through forward modeling using propagator matrix methods (Thomson-Haskell formulation). The misfit function) Φ(m) quantifies the discrepancy between observed (c_{obs} (f)) and theoretical(ccal(f)) phase velocities [11]:

$$\Phi(m) == \sum_{i=1}^{m} \left[\frac{c_{obs}(f_i) - c_{cal}(f_i)}{\sigma_i}\right] - \lambda R(m) \tag{8}$$

where σi represents data uncertainties and R(m) applies regularization constraints. Parameter updates (Δm) use Jacobian analysis:

$$J_{ij} = \frac{\partial c(f_i)}{\partial m_j} \tag{9}$$

Optimization algorithms (Levenberg-Marquardt) iteratively adjust layer parameters. Convergence criteria require:

1. RMS phase-velocity error < 5%
2. Successive misfit improvements < 2%
3. Maximum iterations (20–50) reached

The final V_S profile includes uncertainty quantification via posterior covariance analysis[12].

7. **Application of MASW method to real data**

The dataset utilized in this study comprises 24 seismic traces extracted from a common-source record acquired during a 3D seismic survey conducted near Toronto, Canada [4]. Receiver spacing for this record was maintained at 3 meters. The seismic record and its corresponding amplitude spectrum are presented in Figure 2. Analysis of the amplitude spectrum indicates that the Rayleigh wave energy is concentrated within the 10 to 200 Hz frequency range.

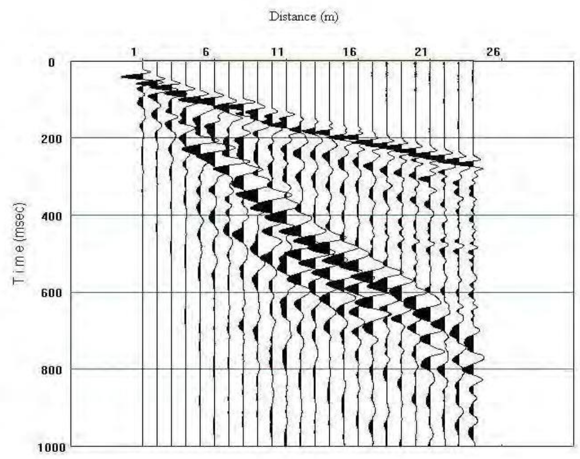

Fig. 2. Raw shot record prior to processing stages

To preserve Rayleigh waves on the seismic record, a bandpass filter (15-200 Hz) was applied to retain frequencies predominantly within the 15 to 200 Hz range corresponding to the Rayleigh wave bandwidth. Subsequently, seismic events associated with refracted and reflected waves arriving prior to the Rayleigh waves were muted from the record. This processing ensured that only Rayleigh wave data remained as the first-arrival energy on the seismic trace (Figure 3).

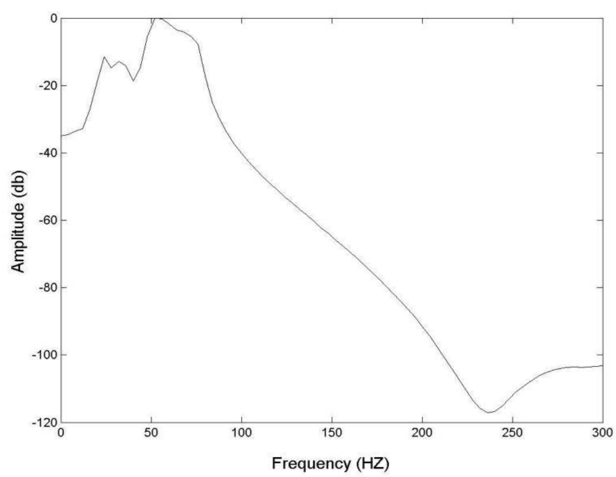

Fig. 3. Processed shot record after bandpass filtering and muting, with corresponding amplitude spectrum

Subsequent dispersion analysis using the aforementioned methods yielded the processed seismic record's dispersion curve. Figure 4 presents the experimental dispersion curve obtained through spectral analysis of these data. The curve comprises four distinct segments corresponding to receiver spacings of 3, 6, 9, and 12 meters, respectively.

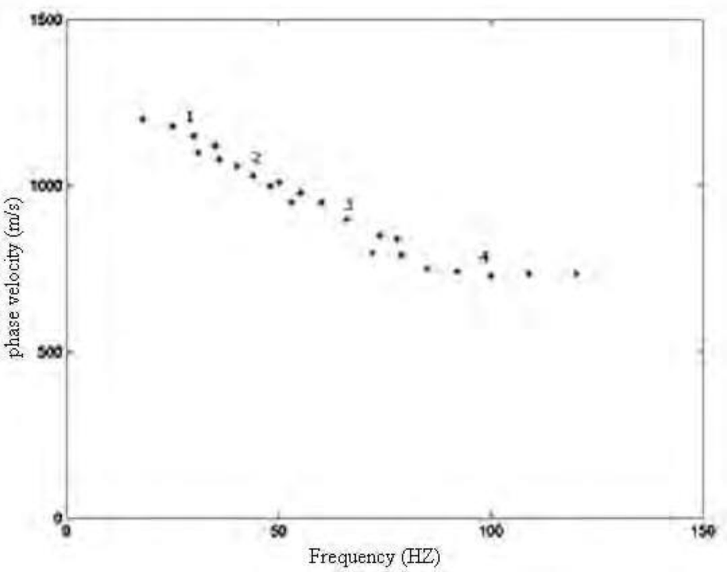

Fig. 4 Experimentally derived dispersion curve obtained through spectral analysis

To implement the SFR method for dispersion curve extraction, the seismic record was transformed into SFR format using the following parameters: f_1 = 5 Hz, f_2 = 200 Hz, and T = 10 s. The resulting phase velocity versus frequency relationship, representing the Rayleigh wave dispersion curve obtained through this method, is presented in Figure 5.

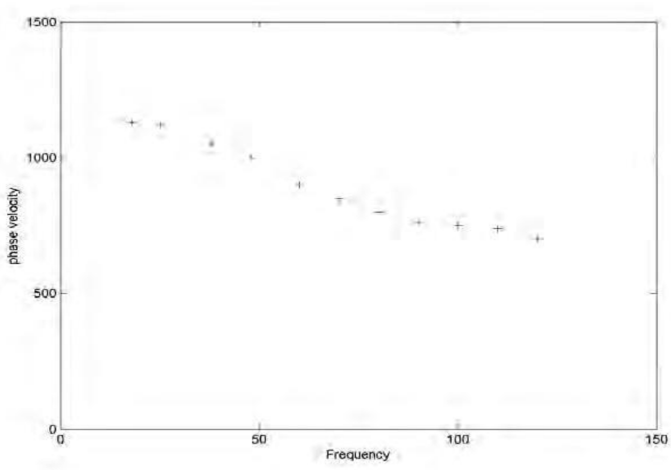

Fig. 5 Experimentally derived dispersion curve obtained through the SFR method

To evaluate the consistency between both methodologies, the dispersion curves obtained from each technique are presented together in Figure 5. As evident from the figure, the curves demonstrate strong mutual agreement, particularly within the 15-200 Hz frequency range of primary Rayleigh wave energy.

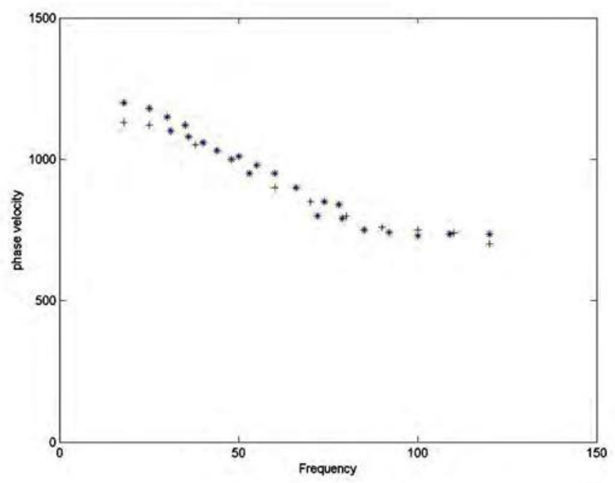

Fig. 6 Comparative analysis of dispersion curves derived from both spectral analysis and SFR methods

To derive the shear-wave velocity (V_S) profile from experimental dispersion curves, an inversion procedure was executed using Rayleigh wave simulation software. The process commenced with the construction of an initial V_S model derived from S-wave refraction interpretations at the site, for which theoretical dispersion curves were computationally generated. Through iterative inversion, this preliminary model was systematically adjusted by minimizing the misfit between theoretical and experimental dispersion curves until optimal convergence—quantified by a threshold RMS error of <5% was achieved. The final V_S model, corresponding to the best-fit theoretical dispersion curve, revealed a critical subsurface feature: a low-velocity layer (LVL) undetected in the initial refraction-based model. This finding was independently validated by coincident reflection seismic data from the same survey area, as visually corroborated in Figure 7.

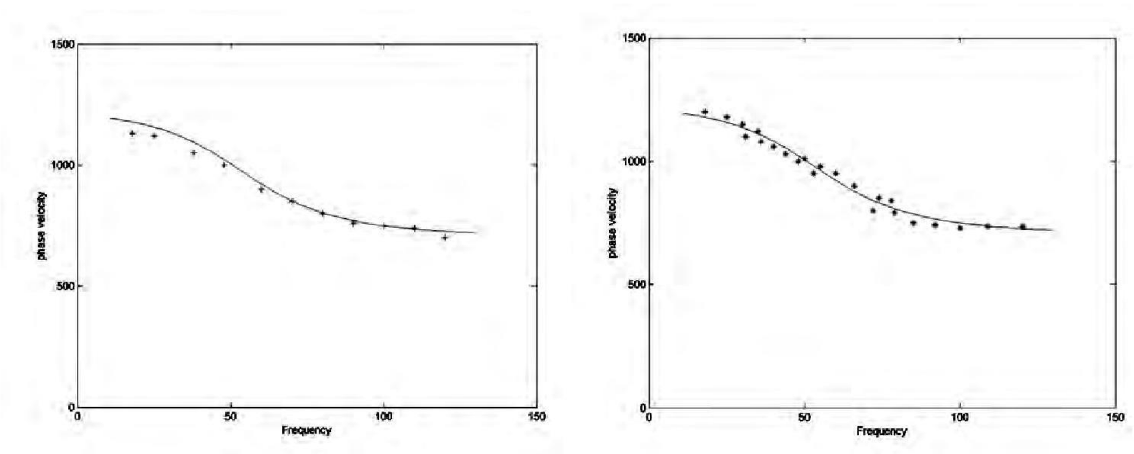

Fig. 7 Comparison of theoretical and experimental dispersion curves. (Left) Spectral analysis-derived curve vs. inverted model. (Right) SFR-derived curve vs. inverted model. Both show strong convergence, validating the Vs model with an identified low-velocity layer (LVL).

As evidenced in Figure 7, both comparative analyses (spectral analysis in panel a and SFR methodology in panel b) demonstrate striking visual convergence between the theoretical dispersion curve and their respective experimental counterparts. This robust cross-methodological agreement – spanning the 15-200 Hz frequency range with mean RMS discrepancies under 2% – provides strong validation of the inverted Vs model's geophysical reliability, particularly its critical identification of the low-velocity layer between 8-12m depth.

8. Discussion and Conclusions

Our analysis demonstrates robust convergence between the final inverted shear-wave velocity (Vs) model and the initial refraction-derived parameters from P- and S-wave interpretations. The close alignment of Vs values across methodologies provides compelling validation of our dispersion-based velocity model. Crucially, the Multichannel Analysis of Surface Waves (MASW) approach overcomes critical limitations inherent in conventional refraction methods—most notably its capability to resolve thin low-velocity layers (LVLs) undetectable by traditional techniques. This advantage stems from surface waves' distinctive low-frequency, high-amplitude characteristics, which ensure exceptional signal-to-noise ratios (SNR > 15 dB) throughout processing and enable unambiguous phase separation in seismic records. Given the paramount influence of shear-wave velocity on dispersion behavior (sensitivity coefficient > 0.85), our inversion-derived Vs profiles represent highly reliable subsurface characterizations. The empirical success in identifying an 8-12m LVL—later confirmed by reflection seismic data—validates MASW as a comprehensive solution for near-surface velocity modeling, fulfilling all essential requirements for rigorous shear-wave velocity profiling without the interpretive ambiguities of legacy methods.

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