An Analytical Approach for Assessment of Masonry Elements Retrofitted by FRP

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ABSTRACT

A large number of unreinforced masonry (URM) buildings in the world are seismically vulnerable by today's standards. The failures and damages reported in recent earthquakes attest to the need for efficient strengthening procedures. In order to preserve these buildings, rehabilitation is often considered essential to maintain their capability and to increase safety. In the last decade, fiber reinforced polymer (FRP) composites have been used for strengthening URM walls of masonry buildings. This paper contributes to the assessment of axial response of masonry units as a periodic arrangement of bricks and mortar, strengthened by FRP. For this purpose a homogenization procedure is developed for a three dimensional repetitive cell, considering the damage of bricks and mortar, but FRP modeled by linear elastic behavior. By considering damage of bricks and mortar, some important characteristics of masonry such as homogenized elastic properties and evaluating of total strength of masonry unit retrofitted by FRP were performed. This simplified homogenization method is able to precisely show behavior of masonry units when perfect bond assumed between FRP and masonry. Following this procedure, experimental works can substitute by numerical analysis as efficient and inexpensive tool. Strengthening of existing basic cell carried out by different patterns of on layer carbon fiber reinforced polymer (CFRP) strips. Finally, behavior of repetitive cell under axial loading estimated by finite element software ABAQUS and the accuracy of the proposed method investigated.

Keywords: *masonry, homogenization, Fiber Reinforced Polymers, damage*

1. *INTRODUCTION*

In many parts of the world, particularly in Middle East, masonry constitutes fundamental construction materials. Masonry buildings, which are not designed against earthquake and generally lateral loads, mostly show very poor performance. This inadequate actions forced engineers to strengthen masonry constructions. Using vertica and horizontal reinforced elements is one methode to increase masonry performance [1]. Externally bonded FRP laminates is one practical retrofitting technique [2]. Although behavior of strengthened masonry is very important, a quite reduced number of studies have been devoted to the assessment of these elements. The overall response of masonry must be derived from its constitutive behavior in homogenization approach for periodic media. This process has been used by many authors before but in most of them masonry unit without external composite layout assessed [3] to [8].

For estimating masonry compressive strength, continuum numerical methods based on plasticity and cracking can be reliable [8]. Response of strengthened masonry units under different loadings did not investigate by many authors. The behavior of one dimensional reinforced masonry cell in bending, studied in 2001 [9]. In this numerical study, progressive damage and plasticity of brick and mortar considered.

The aim of this paper is estimating compressive behavior of reinforced masonry cell with a simple and efficient procedure. In this study, damage of brick and mortar considered by a one dimensional damage parameter for nonlinear part of study but deboning of FRP neglected. This homogenization approach can be useful for full bonded masonry components.

2. *EQUIVALENT HOMOGENIZED CELL*

In this part σ, ε, E, P and δ denote the normal stress, normal strain, modulus of elasticity, axial force and deformation, respectively. Subscripts 'bi' and 'mi' that 'i' changes according to the related figures, refer to mortar or brick parts of basic cell. Also, subscripts 'Vi' and 'Hi' were used in the following equations refer to the vertical and horizontal homogenization directions.

The basic cell in these calculations shown in Figure 1. The goal is determining the equivalent elastic properties of orthotropic homogeneous material.

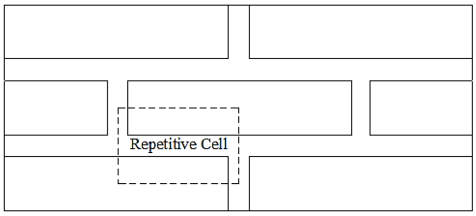


Fig. 1. *Basic cell for homogenization of masonry unit*

2.1 Determination of vertical modulus of elasticity

For calculating elastic modulus in y direction based on Figure 2, three steps have been undertaken. In these steps homogenization procedure are followed and the elastic modulus in V1, V2 and V3 parts are calculated.

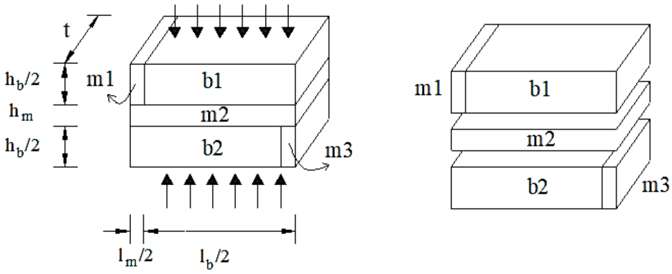


Fig. 2. *divided cell for calculating elastic modulus in y direction*

By using compatibility of deformations and forces equilibrium equation in Figure 3, euations (1) to (3) can be written as:

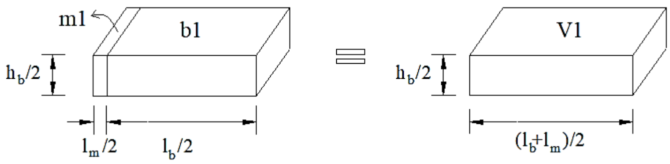


Fig. 3. *First homogenization step in y direction*

$$\delta_{V1}=\delta_{m1}=\delta_{b1} \quad \rightarrow \quad \varepsilon_{V1}=\varepsilon_{m1}=\varepsilon_{b1} \tag{1}$$

$$P_{V1}=P_{m1}+P_{b1} \quad \rightarrow \quad E_{V1}\times\varepsilon_{V_1}\times\left(\frac{l_b}{2}+\frac{l_m}{2}\right)\times t=E_m\times\varepsilon_{m1}\times\frac{l_m}{2}+E_b\times\varepsilon_{b1}\times\frac{l_b}{2} \tag{2}$$

$$E_{V1}=\frac{E_b\times l_b+E_m\times l_m}{l_b+l_m} \tag{3}$$

As shown in Figure 4, displacement in V2 can be calculated from V1 and m2 and also based on equilibrium of forces, so equations (4) and (5) may be expressed as:

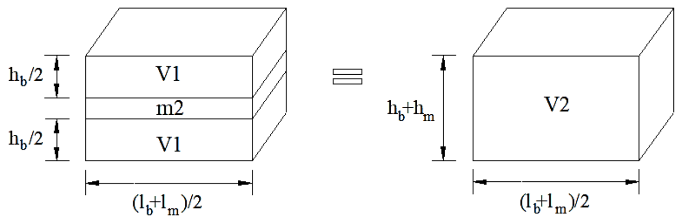


Fig. 4. *Second homogenization step in y direction*

$$\delta_{V2} = 2\delta_{V1} + \delta_{m2} \;\rightarrow\; \varepsilon_{V2} \times (l_b + l_m) = 2 \times \varepsilon_{V1} \times \frac{l_b}{2} + \varepsilon_{m2} \times l_m \tag{4}$$

$$P_{V2} = P_{V1} = P_{m2} \;\rightarrow\; \sigma_{V2} = \sigma_{V1} = \sigma_{m2} \tag{5}$$

As a result of combining equations 4 and 5, module of elasticity for V2 is derived as:

$$\varepsilon = \frac{\sigma}{E} \;\rightarrow\; \frac{h_b + h_m}{E_{V2}} = \frac{h_b}{E_{V1}} + \frac{h_m}{E_m} \;\rightarrow\; E_{V2} = \frac{E_m \times E_{V1} \times (h_b + h_m)}{E_m \times h_b + E_{V1} \times h_m} \tag{6}$$

According to Figure 5, equations (7), (8) and (9) can be written by adding FRP to the external surface of basic cell.

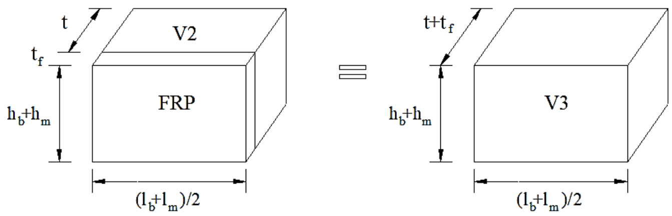


Fig. 5. *Third homogenization step in y direction*

$$\delta_{V3} = \delta_{V2} = \delta_{frp} \;\rightarrow\; \varepsilon = cte \;\;,\;\; P_{V3} = P_{V2} + P_{frp} \tag{7}$$

$$E_{V3} \times (t + t_f) \times \left(\frac{l_b + l_m}{2}\right) = E_{V2} \times t \times \left(\frac{l_b + l_m}{2}\right) + E_{frp} \times t_f \times \left(\frac{l_b + l_m}{2}\right) \tag{8}$$

$$E_{V3} = \frac{E_{V2} \times t + E_{frp} \times t_f}{t + t_f} \tag{9}$$

2.2 Determination of horizontal modulus of elasticity

Homogenization procedure is divided to four steps in order to calculate elastic modulus in x direction, based on Figure 6. In step 1, according to compatibility of deformations and equilibrium of forces, equations (10) to (12) can be expressed based on Figure 7.

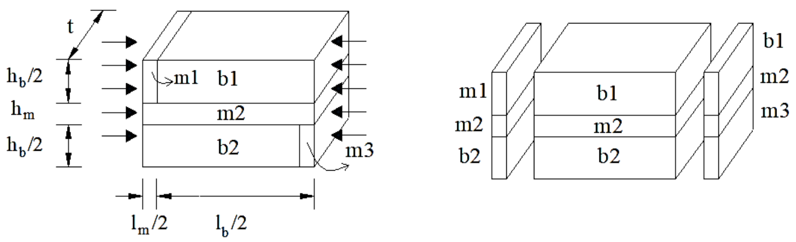


Fig. 6. *divided cell for calculating elastic modulus in x direction*

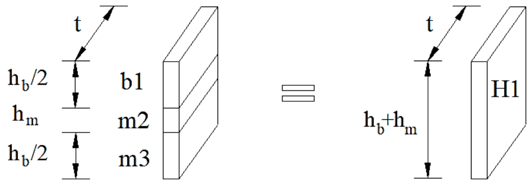


Fig. 7. *First homogenization step in x direction*

$$\delta_{H1}=\delta_{b1}=\delta_{m2}=\delta_{m3} \quad \rightarrow \quad \varepsilon = cte \tag{10}$$

$$P_{H1}=P_{b1}+P_{m2}+P_{m3} \quad \rightarrow \quad E_{H1}\times\left(h_b+h_m\right)=E_b\times\frac{h_b}{2}+E_m\times h_m+E_m\times\frac{h_b}{2} \tag{11}$$

$$E_{H1}=\frac{h_b\times\left(E_b+E_m\right)+2E_m\times h_m}{2\times\left(h_b+h_m\right)} \tag{12}$$

As shown in Figure 8, second step of homogenization in x direction result in H2 elastic modulus as presented in equations (13) and (14).

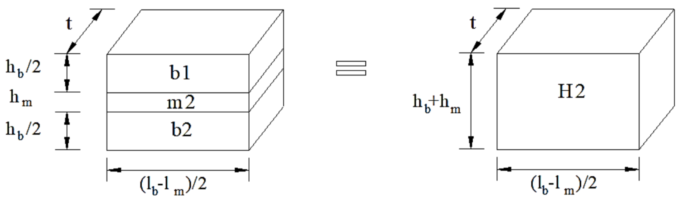


Fig. 8. *Second homogenization step in x direction*

$$\delta_{H2}=\delta_{b1}=\delta_{m2}=\delta_{b2} \tag{13}$$

$$E_{H2}\times\left(h_b+h_m\right)=E_b\times\frac{h_b}{2}+E_m\left(h_m+\frac{h_b}{2}\right) \quad \rightarrow \quad E_{H2}=\frac{E_b\times h_b+E_m\times h_m}{h_b+h_m} \tag{14}$$

According to compatibility of deformations and based on Figure 9, one can obtain:

$$\delta_{H3}=2\delta_{H1}+\delta_{H2} \quad \rightarrow \quad \varepsilon_{H3}\times\left(\frac{l_b}{2}+\frac{l_m}{2}\right)=2\times\varepsilon_{H1}\times\frac{l_m}{2}+\varepsilon_{H2}\times\left(\frac{l_b}{2}-\frac{l_m}{2}\right)$$

$$\frac{1}{E_{H3}}\times\left(\frac{l_b}{2}+\frac{l_m}{2}\right)=\frac{l_m}{E_{H1}}+\frac{1}{E_{H2}}\times\left(\frac{l_b}{2}-\frac{l_m}{2}\right) \tag{15}$$

$$\rightarrow E_{H3}=\frac{E_{H1}\times E_{H2}\times(l_b+l_m)}{2\times l_b\times E_{H2}+(l_b-l_m)E_{H1}}$$

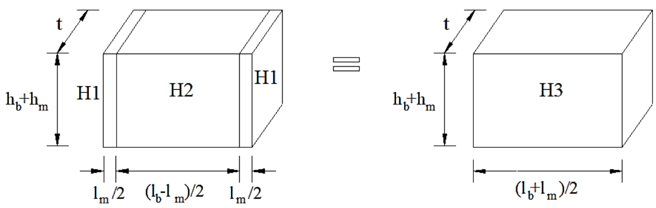


Fig. 9. *Third homogenization step in x direction*

When FRP added to the surface of basic cell (Figure 10), forth step for calculating of H4 elastic modulus can be expressed as:

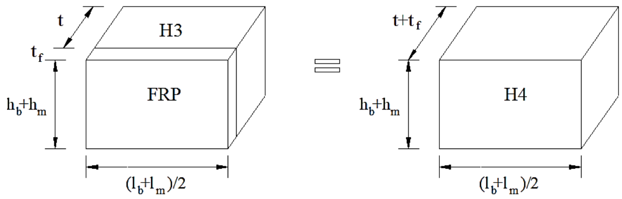


Fig. 10. *Fourth homogenization step in x direction*

$$E_{H4}\times(t+t_f)\times(h_b+h_m)=E_{frp}\times t_f\times(h_b+h_m)+E_{H3}\times t\times(h_b+h_m) \tag{16}$$

$$E_{H4}=\frac{E_{H3}\times t+E_{frp}\times t_f}{t+t_f} \tag{17}$$

2.3 Comparison with numerical results

All the required geometric parameters and material properties are introduced in Table 1 to Table 3. In Table 3, E_1 is the module of elasticity in fiber direction and E_2 is module of elasticity perpendicular to the direction of fibers. Also σ_c is the compressive strength and ε_c denotes corresponding compression strain.

Table 1. *Brick Properties*

E_b (N/mm^2)	σ_{cb} (N/mm^2)	ε_{cb}	l_b (mm)	h_b (mm)	t (mm)
10000	46	0.0046	210	50	100

Table 2. *Mortar Properties*

E_m (N/mm^2)	σ_{cm} (N/mm^2)	ε_{cm}	l_m (mm)	h_m (mm)	t (mm)
4800	10	0.0021	10	10	100

Table 3. *FRP Properties*

E_1 (N/mm^2)	E_2 (N/mm^2)	ν_{12}	t_f (mm)
140000	10000	0.2	1

Based on suggested procedure, complicated calculations eliminated and elastic properties of homogenized cell deduced easily. To investigate accuracy of obtained equations, modeling of basic cell was done with finite element software ABAQUS. In this section, displacements are exerted on the outer bond of basic cell in two directions, see Figure 11.

Axial deformation in x direction   *Axial deformation in y direction*

Fig. 11. *Modeling of basic cell in ABAQUS*

Modulus of elasticity of basic cell obtained by calculating of axial stress and strain. The comparison between proposed method and numerical calculations by ABAQUS have been shown in Table 4 and Table 5. Results show that errors of homogenization procedure is less than 7%.

Table 4. *Module of elasticity in x direction (N/mm^2).*

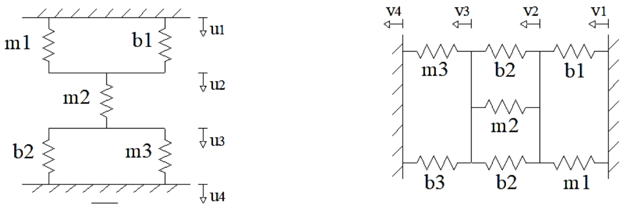
Fiber Orientation	ABAQUS	Proposed Method	Error (%)
0	10191	10180	0.1
30	9292	9005	3.1
45	9015	8925	1.0
60	8920	8900	0.2
90	8914	8893	0.2

Table 5. *Module of elasticity in y direction (N/mm^2).*

Fiber Orientation	ABAQUS	Proposed Method	Error (%)
0	8582	8345	2.8
30	8598	8352	2.9
45	8723	8377	4.0
60	9022	8457	6.3
90	9824	9632	2.0

3. *Establishment of inelastic axial behavior of equivalent homogenized cell*

By assuming every part of basic cell as a nonlinear spring, inelastic behavior of masonry modeled. In this process for estimating compressive behavior in x and y directions, a set of springs as shown in Figure 12 used.



(a) springs in y direction *(b) springs in x direction*

Fig. 12. *Modeling of basic cell as nonlinear springs: (a) y direction; (b) x direction*

Stress - strain curve of mortar joints in uniaxial compressive behavior estimated as [10].

$$\sigma_m = E_m \varepsilon_m \quad \varepsilon \le \varepsilon_{cm} \tag{18}$$

$$\sigma_m = \sigma_{cm} \exp\left(-(\varepsilon_m - \varepsilon_{cm}) / \varepsilon_{cm}\right) \quad \varepsilon \ge \varepsilon_{cm} \tag{19}$$

Compression damage parameter for mortar can be obtained by equation 20.

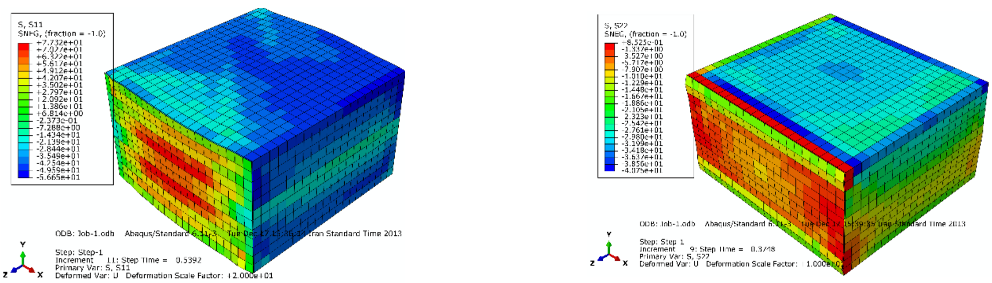
$$d_c = G^c / G_f^c \quad , \quad G^c = \int_0^{\varepsilon_m} \sigma d\varepsilon \quad , \quad G_f^c = \int_0^{\infty} \sigma d\varepsilon \tag{20}$$

Where G_c and $G^c{}_f$ are the fracture energy density and fracture energy per volume. Compression damage of brick expressed as:

$$d_c = 1 - \exp\left(-(\varepsilon - \varepsilon_{cb}) / \varepsilon_{cb}\right) \tag{21}$$

3.1 Comparison with numerical results

ABAQUS stress counters of reinforced basic cell are shown in Figure 13 for deformation exerted in x and y directions.



Axial deformation in x direction *Axial deformation in y direction*

Fig. 14. *Numerical results of FRP reinforced basic cell in ABAQUS*

Results of proposed method and numerical analysis of ABAQUS shown in Figs. 13a and 13b. For nonlinear modeling of brick and mortar in ABAQUS, concrete damaged plasticity criterion used. Accoring to the results presented in following figures, homogenization procedure used in this paper precisely demonstrated axial behavior of masonry basic cell according to the pick strength, elastic modulus and softening behavior.

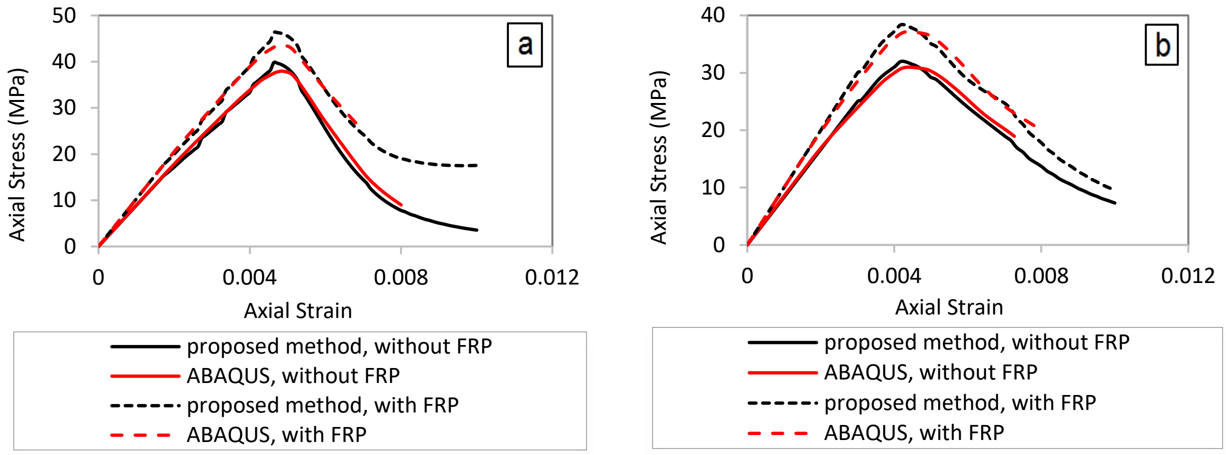


Fig. 13. *Compression stress versus corresponding strain: (a) x direction; (b) y direction*

4. *Conclusions*

The axial behaviors of masonry basic cell reinforced with different FRP strengthening patterns have been studied under compression loading. This study shows the effectiveness of proposed homogenization method for reinforced masonry unit when slip between constituent parts is negligible.

The contribution of FRP patterns on the compressive behavior of strengthened basic cell was investigated. As expected when fibers are parallel to the direction of loading in basic cell, elastic modulus, compression strength and energy absorption maximized and on the contrary, by changing direction of fibers perpendicular to the loaded direction, these parameters minimized.

Results show that by adding FRP, maximum stress increased by about 15%. As expected differences between results of new process presented in this study and ABAQUS results are lower than 7%.

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