Analysis of Operational Costs for Navigation Satellite Constellations Based on LEO and MEO Orbits with the Geometric Dilution of Precision (GDOP) Criterion

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ABSTRACT

Satellite constellations have become a major topic in space engineering due to their impact on the economy, society, and military affairs. The reduction in launch and satellite design costs has led private companies to compete in building and designing satellite constellations with various missions. The operational costs for these satellites will be analyzed based on their arrangement in LEO and MEO orbits. In this study, cost estimation will be based on the type of satellite and their launch costs using local launch vehicles. In the first scenario, with 90 satellites arranged symmetrically at a 55-degree inclination in LEO at an altitude of 1,000 km, a GDOP of 3.92 and a maximum operational cost of $450 million will be achieved. In the second scenario, with 24 satellites in MEO at an altitude of 20,000 km, a GDOP of 2.6 and a maximum operational cost of $9.6 billion are anticipated.

Keywords: satellite, satellite constellations, cost of launching,navigation,

1. *INTRODUCTION*

Satellite navigation constellations began with the launch of the Transit system by the United States in the 1960s, originally designed for naval navigation. This was followed by the development of GPS (Global Positioning System) in the 1970s, providing global navigation capabilities for both military and civilian applications. Inspired by GPS, other countries launched their own systems, including GLONASS by Russia, Galileo by the European Union, and BeiDou by China. These constellations have evolved over decades, enhancing positioning accuracy and coverage worldwide [1]. Satellite constellations are based on the different missions they can perform while being the lifestyle of humans. The change in the speed of information transmission, satellite networks, the use of satellite internet and navigation has caused the space economy to grow significantly[2].

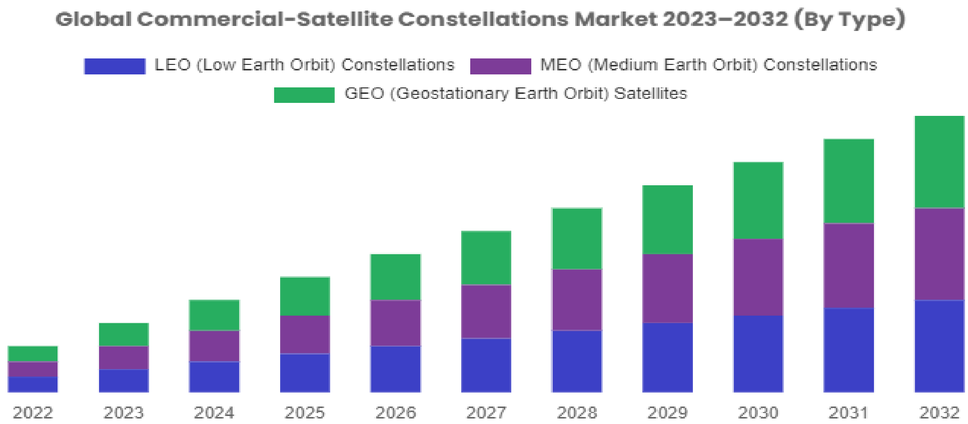

Fig.1. the global commercial satellite constellations market size is expectd to record a CAGR %13.09 from 2023 to 2032.

In 2022, the market size is projected to reach a valuation of USD 12.6 billion. By 2032, the valuation is anticipated to reach USD 43.1 billion [3]. The work done shows that today the issue of navigation satellite constellations is turning into an economic issue to increase the efficiency of basic space navigation. For this reason, the problem of the cost of designing and implementing a navigation satellite constellation should be carefully studied. The cost of launching a navigation satellite into Medium Earth Orbit (MEO) can vary significantly based on factors such as payload weight, launch vehicle, and the launch provider. Generally, the expense ranges from $50 million to $400 million per satellite. For example, the GPS III satellites, part of the U.S. GPS constellation, are estimated to have an average cost of $500 million each, which includes the satellite build and launch expenses. Modern advancements, such as reusable rockets, have the potential to reduce these costs over time [4]. But on the other hand, The cost of launching a navigation satellite into Low Earth Orbit (LEO) is generally lower than launching into higher orbits, with average launch expenses ranging from $10 million to $90 million, depending on the launch provider, satellite specifications, and payload mass. For instance, small LEO satellites, like those in certain commercial constellations, can cost as low as $1 million to $5 million per satellite when launched in clusters. Advances in reusable rocket technology, such as those developed by SpaceX, have further contributed to reducing launch costs [5]. In addition to the costs of designing and implementing a navigation satellite constellation, it must successfully execute the mission it has been designed for. One of the criteria for designing a satellite constellation is the Geometric Dilution of Precision(GDOP), based on which the designers try to arrange the satellites. to do in such a way that the satellite constellation can provide its navigation services with proper accuracy. Complementary solutions to the Medium Earth Orbit (MEO) Global Navigation Satellite Systems (GNSS) are more and more in demand to be able to achieve seamless positioning worldwide, in outdoor as well as in indoor scenarios, and to cope with increased interference threats in GNSS bands [6]. Two of such complementary systems can rely on the emerging Low Earth Orbit (LEO) constellations and on the terrestrial long-range Internet of Things (IoT) systems, both under rapid developments nowadays [7]. The design of satellite constellations is based on various parameters such as mission, cost and orbital configuration. avigation satellite constellations, such as GPS and Galileo, rely on carefully designed orbital configurations to ensure global coverage and high accuracy. Commonly used configurations include Walker and Flower constellations, which allow for optimized satellite placement to minimize gaps in coverage and enhance reliability. These arrangements ensure that users worldwide have access to a sufficient number of satellites for precise positioning [8]. Recent studies also highlight the benefits of inclined and elliptical orbits to improve accuracy in specific regions [9]. Navigation constellations in LEO (Low Earth Orbit) provide lower latency and faster communication speeds, making them suitable for real-time applications and high-accuracy positioning. However, due to their lower altitude, LEO satellites have a limited field of view, requiring a larger number of satellites to achieve global coverage. In contrast, MEO (Medium Earth Orbit) constellations, such as GPS and Galileo, offer broader coverage with fewer satellites, benefiting from higher orbital altitudes, but they experience slightly higher latency and signal travel time [10]. tudies suggest that hybrid constellations combining both LEO and MEO satellites could optimize coverage, accuracy, and resilience in navigation services. In this paper, the design of a navigation satellite constellation in LEO and MEO is addressed, considering circular and elliptical orbits, and a comparison of costs, number of launches, and number of satellites will be made based on geometric dilution of precision, using the minimum number of satellites arranged symmetrically.

2. *Orbital arrangement of the satellite*

The orbital arrangement of satellites refers to the specific positioning and configuration of satellites in their designated orbits to achieve desired coverage and performance. Different types of constellations, such as Walker, polar, and inclined arrangements, are used based on mission objectives. For example, polar orbits allow satellites to cover the entire Earth as the planet rotates, making them ideal for global observation missions, while inclined orbits focus on specific regions. Proper orbital arrangement ensures optimal satellite distribution, minimizes gaps in coverage, and helps manage costs by reducing the total number of satellites needed.

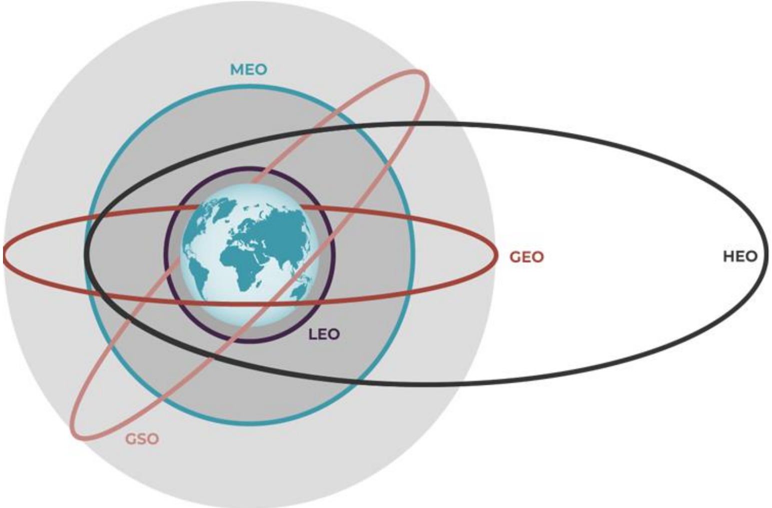

Fig. 2. Classification of satellite orbits

LEO (Low Earth Orbit): Ranges from 160 to 2,000 km above Earth, often used for Earth observation, communications, and some navigation satellites due to low latency and fast orbits. MEO (Medium Earth Orbit): Typically between 2,000 and 35,786 km altitude; ideal for navigation constellations like GPS and Galileo, balancing coverage and signal latency. GEO (Geostationary Earth Orbit): Positioned at approximately 35,786 km, GEO satellites match Earth's rotation, providing continuous coverage over a fixed area, commonly used for weather, TV broadcasting, and some communications. HEO (Highly Elliptical Orbit): Has an elongated elliptical shape, reaching high altitudes at apogee; suitable for providing long-duration coverage over high-latitude regions, often used for specialized communication and reconnaissance.

3. Geometric Dilution of Precision (GDOP)

Dilution of precision (DOP), or geometric dilution of precision (GDOP), is a term used in satellite navigation and geomatics engineering to quantify how the geometry of satellites in view impacts the precision of positional measurements. GDOP describes the relationship between satellite positions and the error propagation in calculating a user's position. It indicates how errors in the satellite signals—such as timing inaccuracies or atmospheric interference—translate into inaccuracies in the user's position.A lower DOP value signifies a better satellite geometry and more accurate positioning, while higher DOP values indicate poor satellite geometry, leading to increased error margins. For instance, satellites evenly distributed across the sky provide a low DOP (high accuracy), while satellites clustered in one part of the sky yield a high DOP (low accuracy). The concept of DOP originated with the Loran-C navigation system but has become integral to satellite navigation systems like GPS, Galileo, and GLONASS. These systems use the GDOP metric to assess and optimize their performance, ensuring precise and reliable positioning for applications such as aviation, marine navigation, and geospatial data collection.

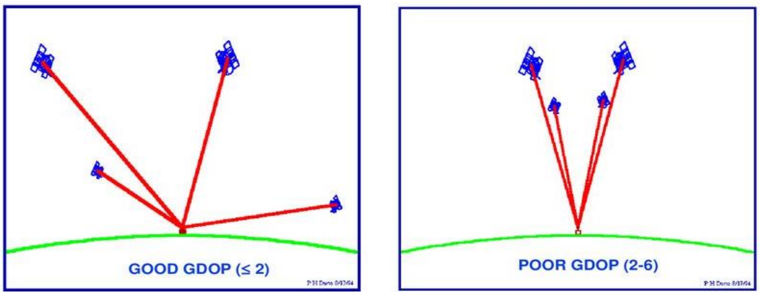

Fig.*3.* How to arrange the satellites and its effect on the GDOP

Geometric Dilution of Precision (GDOP) is a measure used to quantify how satellite geometry impacts the accuracy of a navigation solution. GDOP is derived from the satellite-to-user geometry matrix A:

$$A=\begin{bmatrix} \frac{(x_1-x)}{R_1} & \frac{(y_1-y)}{R_2} & \frac{(z_1-z)}{R_3} & -1 \\ \frac{(x_2-x)}{R_2} & \frac{(y_2-y)}{R_2} & \frac{(z_2-z)}{R_2} & -1 \\ \frac{(x_3-x)}{R_3} & \frac{(y_3-y)}{R_3} & \frac{(z_3-z)}{R_3} & -1 \\ \frac{(x_4-x)}{R_4} & \frac{(y_4-y)}{R_4} & \frac{(z_4-z)}{R_4} & -1 \end{bmatrix} \quad (1)$$

where the GDOP value is calculated from the trace of the covariance matrix:

$$Q_{dop}=(A^T A)^{-1} \quad (2)$$

The Qdop matrix is approximated from the covariance matrix of the unknown noises of the measured values. This approximation is obtained by the First Order Second Moment FOSM, which is a method to quantify the uncertainty.

$$Q=\begin{bmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{xz} & \sigma_{xt} \\ \sigma_{yx} & \sigma_y^2 & \sigma_{yz} & \sigma_{yt} \\ \sigma_{zx} & \sigma_{zy} & \sigma_z^2 & \sigma_{zt} \\ \sigma_{tx} & \sigma_{ty} & \sigma_{tz} & \sigma_t^2 \end{bmatrix} \quad (3)$$

where σx, σy, σz, and σt represent the positional uncertainties in the x, y, z coordinates, and time. Lower GDOP values indicate better satellite geometry and improved positioning accuracy.

$$GDOP=\sqrt{\sigma_x^2+\sigma_y^2+\sigma_z^2+\sigma_t^2}=trace(Q_{dop}) \quad (4)$$

4. simulation

To design a navigation satellite constellation, two orbits of Leo and Meo are considered. The altitude in the Leo orbit is considered 1000 km and in the Meo orbit it is considered 20000 km. Satellite orbits are considered symmetrical at an inclination angle of 55 degrees and symmetrical in different orbital planes.

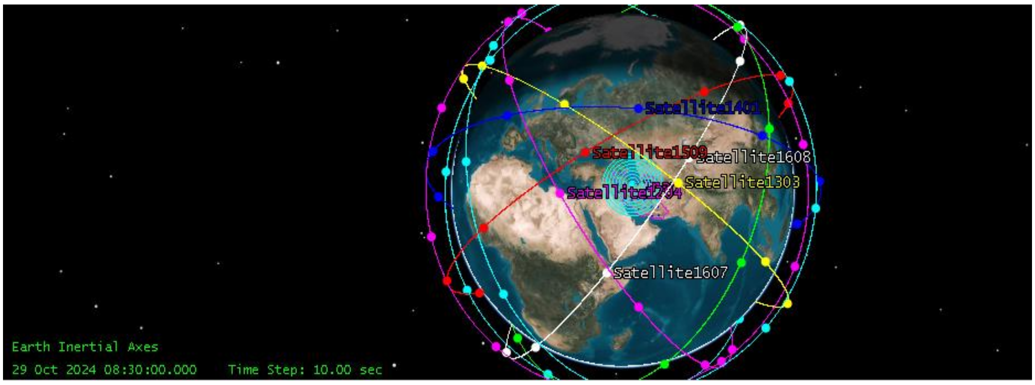

***Fig.*4.** 3D design of navigation satellite constellation in Leo orbit

The orbits in the Leo orbit are considered circular and the simulation is considered for a 24-hour period.

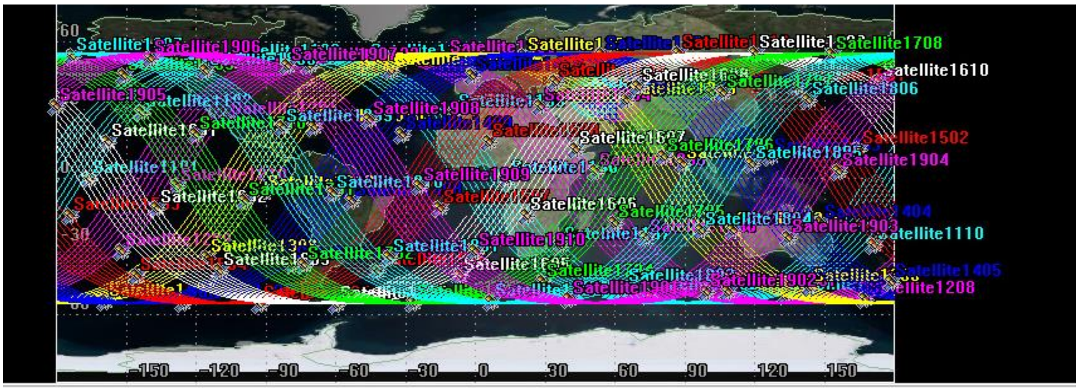

***Fig.*5.** Two-dimensional simulation of navigation satellite constellation in Leo orbit

By designing a navigation constellation in the Leo orbit, it is now possible to extract information related to the GDOP

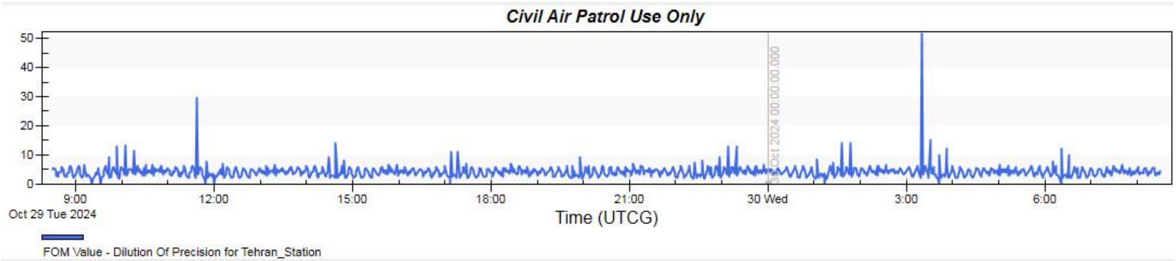

Fig.6. GDOP value in 24 hours for the constellation of satellite navigator in Leo orbit

. As it is clear in the figure of Shish number, in some seconds, due to the weak arrangement, the GDOP is weak, but in general, the average GDOP in 24 hours is 3.92, which is considered a good value.

***Table 1**. Navigation constellation in the first scenario*

circular	*Circuit type*
90	*Number of satellites*
9	*number of orbital planes*
1000km	*Altitude*
55 deg	*inclination angle*
3.92	*average GDOP value over 24 hours*
90-450 million dollars	*operational cost*

Now in Meo orbit with 24 satellites in 6 symmetrical orbital planes,It is planned to simulate a navigation satellite constellation. The reason for the arrangement's symmetry is to reach the optimal GDOP.

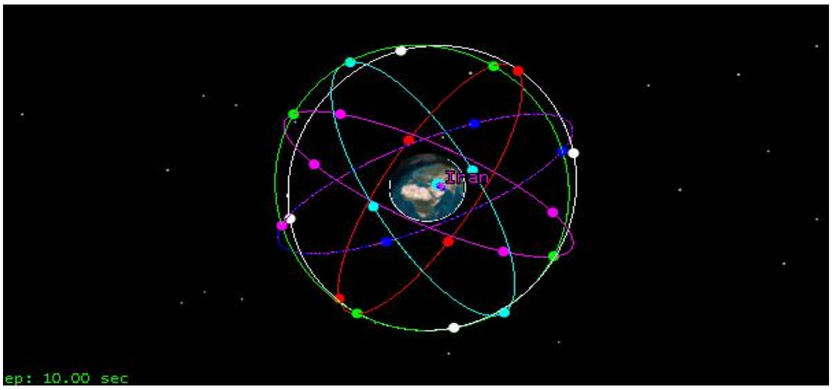

Fig.7. *3D display of satellite constellation in Meo orbit*

At an inclination angle of 55 degrees, based on the launch position of the glass, a symmetrical orbital plane is arranged.

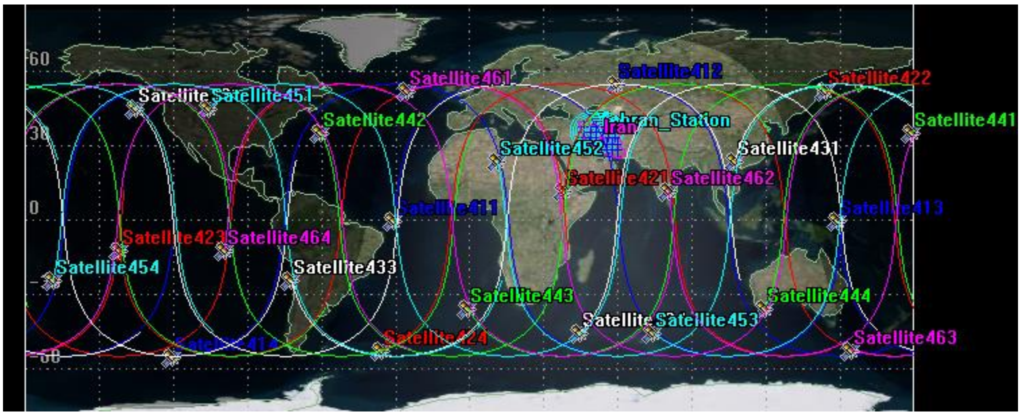

***Fig.*8.** Two-dimensional display of navigation satellite constellation in Meo orbit

Now it is possible to extract GDOP changes based on time.

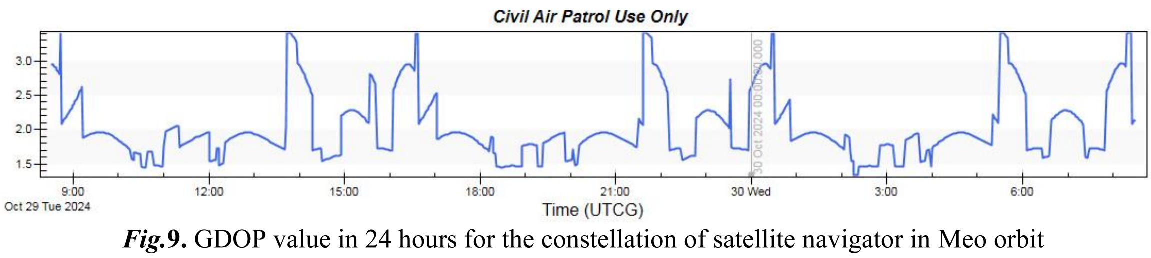

***Fig.*9.** GDOP value in 24 hours for the constellation of satellite navigator in Meo orbit

As it is known, the amount of G-Dap has improved far compared to Leo's orbit and the average change is 2.6 On the other hand, the maximum change is 3.4 and the minimum change is 1.4.

***Table 2**. Navigation constellation in the first scenario*

circular	*Circuit type*
24	*Number of satellites*
6	*number of orbital planes*

20000km	*Altitude*
55 deg	*inclination angle*
2.6	*average GDOP value over 24 hours*
1.2-9.6 billion dollars	*operational cost*

Despite the efficiency of the constellation of navigation satellites in the MEO orbit, the cost of implementing and making it operational is far more than that of the Leo orbit.

5. conclusion

In this paper, the design of navigation satellite constellations in LEO and MEO orbits was investigated. Two scenarios were then developed for MEO and LEO altitudes, considering an inclination angle of 55 degrees over Iran, resulting in the design of two satellite navigation constellations for the Iranian region. The performance criterion for these constellations is the Geometric Dilution of Precision (GDOP). According to the design in the LEO orbit with 90 satellites across nine orbital planes, a navigation satellite constellation with a GDOP of 3.92 can be achieved, with a maximum operational cost of $450 million based on existing cost models. In the second scenario, a navigation satellite constellation in MEO orbit was designed, achieving a GDOP of 2.6 with 24 satellites. However, the operational cost for this MEO-based constellation is estimated to be $9.6 billion. It appears that with optimized constellation design in LEO, a more efficient and cost-effective navigation satellite constellation can be implemented.

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