Investigation of Empirical Prediction Equations for Maximum Scour Depth Under a Single Pipeline in River Flows

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ABSTRACT

River flows could form a local scour hole beneath the pipelines which transporting fluids in the bed. The scour hole influences the safe performance of these infrastructures and may cause some damage. Therefore, a prediction of the maximum scour hole has been a necessary task. This study examines the empirical equations derived from laboratory studies presented for calculating scour hole depth. For this purpose, data from the physical tests of the laboratory model of a single pipeline scour in the riverbed were collected. Then, the accuracy of the empirical equations in the prediction of the maximum scour depth was examined and compared. To improve the accuracy of the empirical equations, considering their simple applications, the modified equations were developed by similar forms of the empirical equations by keeping the effective parameters constant, and changing the constant coefficients. The computed values of the statistical indices R, RMSE, MAPE, and SI revealed the acceptable improvement of the developed equations in the prediction of the maximum scour hole depth below pipelines in a river environment under unidirectional flow.

Keywords: *Rivers, Pipelines, Experimental models, Scouring, Prediction equations.*

1. *INTRODUCTION*

Pipelines are sometimes installed in river flows to convey fluids such as water, gas, petroleum, etc., on river beds [1]. The scour process is the result of an interaction between water and structure when these pipes are installed directly on the bed or with supports on it. In this case, the pipeline is an obstacle to the river flow, and local scour happens. It leads to changes in flow patterns and the bed morphology disturbances. In the continuous proceed the local scour hole may form under the pipe, and the length of the suspended part of the pipe increases over time [2].

Several studies investigated the scour model under a pipeline in a riverbed with laboratory physical models. Moncada-M and Aguirre-Pe [1] performed an experimental study on the scour hole below a pipe on two varied sizes of bed particles under unidirectional flow. Based on their experimental results, they proposed Eq. (1) and Eq. (2) to calculate the maximum scour depth below pipelines [1]:

$$\frac{d_s}{D} = \tanh(1.4Fr) + 0.55 \quad (1)$$

$$\frac{d_s}{D} = 2Fr\,\mathrm{sech}(1.7\frac{e}{D}) \quad (2)$$

where d_s is the maximum scour depth below the pipeline, D is the pipe diameter, Fr is the flow Froude number ($Fr=\frac{V_0}{\sqrt{gy_0}}$), y_0 is the approaching flow depth, U_0 is the mean velocity of the approaching flow, g is the gravitational acceleration, and e is the gap between the bottom of the pipe and the initial bed level.

Hosseini et al. [3] studied the local scour under pipelines under steady flow using numerical and physical models. They considered the effects of variables such as pipe diameter, water depth, and flow velocity on maximum scour depth [3]. Dey and Singh [4] via experimental tests concluded that the pipeline scour depth depends largely on the ratio of the pipe diameter (D) to the median size of the sediment particles (d_{50}). The scour depth increases by increasing D/d_{50} [4]. Dey and Singh [5] investigated the effect of upward seepage on pipeline scour using the laboratory model. They observed that upward seepage decreases the shear stress and reduces the maximum scour depth of the pipeline. Azamathulla et al. [6] conducted an experimental study of the scour rate in pipelines in rivers bed with angles of 30, 60, and 90 degrees relative to the flow direction. They concluded that the scour depth reduces by varying the angle of deviation from 90 to 30 degrees [6].

Haj-Mohammadi et al. [7] performed experimental tests for single and double pipes under unidirectional flow. They observed that, when water flows around the pipe, the scour process starts by establishing the piping flow from the pipe's upstream side to the downstream side [7]. Pourmirza and Kamanbedast [8] obtained experimental data in the laboratory for a scour below pipelines in rivers. They found that the Shields parameter has a very high effect (more than 95%), determined as one of the most effective causes of local scour.

Some traditional empirical equations are useful in technical literature. The maximum scour depth below pipelines can be computed by Eq. (3) proposed by Kjeldsen et al. [9] as:

$$\frac{d_s}{D} = 0.929\left(\frac{V_0}{2gD}\right)^{0.2} \quad (3)$$

Bijker and Leeuwestein [10] suggested Eq. (4) to predict d_s as:

$$\frac{d_s}{D} = 0.929\left(\frac{V_0}{2gD}\right)^{0.2} D^{-0.21} d_{50}^{-0.04} \quad (4)$$

Where d_{50} is the median sediment size. Ibrahim and Nalluri [11] developed empirical equations for clear water scour below pipelines with Eq. (5) based on the critical velocity (V_c).

$$\frac{d_s}{D} = 4.706\left(\frac{V_0}{V_c}\right)^{0.89}\left(\frac{V_0}{gy_0}\right)^{1.48} + 0.06 \quad (5)$$

It was concluded from previous studies that the accuracy of empirical equations in predicting the maximum scour hole depth under pipelines in the river needs to be evaluated. The present study provides a comprehensive analysis of the accuracy of empirical equations in calculating the maximum scour hole depth under pipelines in rivers using available laboratory data. Also, modified equations were developed to improve the accuracy of empirical equations and were investigated by calculating statistical indices.

2. *METHODOLOGY*

To investigate the maximum depth of scour hole under a pipeline in a riverbed, a comprehensive data set of laboratory results is required. For this study, 5 available datasets from various laboratory studies were used, including Moncada-M and Aguirre-Pe [1], Hosseini et al. [3], Dey and Singh [4], Haj-Mohammadi et al. [7], and Pourmirza and Kamanbedast [8]. Table 1 provides general characteristics of these data, including: the number of data, the type of physical model, the flow type, and the scour type. All data were for a single pipeline scour condition under unidirectional steady flow.

Table 1. *Experimental models used in the present study*

No.	Study	Data	Pysical Model	Flow Type
1	Moncada-M and Aguirre-Pe [1]	90	Single pipeline	Unidirectional steady flow
2	Hosseini et al. [3]	6	Single pipeline	Unidirectional steady flow
3	Dey and Singh [4]	155	Single pipeline	Unidirectional steady flow
4	Haj-Mohammadi et al. [7]	5	Single pipeline	Unidirectional steady flow
5	Pourmirza and Kamanbedast [8]	54	Single pipeline	Unidirectional steady flow

Table 2 presents available empirical prediction equations to investigate the accuracy in the prediction of maximum scour depth below pipelines under unidirectional flow. The equations have been selected from the *studies of Kjeldsen et al. [9], Bijker and Leeuwestein [10], Ibrahim and Nalluri [11], and Moncada-M and Aguirre-Pe [1] (two equations), as Eq. (3), Eq. (4), Eq. (5), Eq. (1), and Eq. (2), respectively. Eq. (3) has V_0/D as the input parameter, Eq. (4) uses D, V_0, and d_{50}. Eq. (5) used V_0/V_c, V_0/y_0 as input parameters. Eq. (1) and Eq. (2) presented with* Fr *and* Fr*, e/D as input parameters, respectively.*

***Table 2**. Empirical prediction equations used in the present study*

No.	Study	Equation No.	Equation
1	Kjeldsen et al. [9]	Eq. (3)	$\frac{d_s}{D}=0.929(\frac{V_0}{2gD})^{0.2}$
2	Bijker and Leeuwestein [10]	Eq. (4)	$\frac{d_s}{D}=0.929(\frac{V_0}{2gD})^{0.2}D^{-0.21}d_{50}^{-0.04}$
3	Ibrahim and Nalluri [11]	Eq. (5)	$\frac{d_s}{D}=4.706(\frac{V_0}{V_c})^{0.89}(\frac{V_0}{gy_0})^{1.48}+0.06$
5	Moncada-M and Aguirre-Pe [1]	Eq. (1)	$\frac{d_s}{D}=\tanh(1.4Fr)+0.55$
6	Moncada-M and Aguirre-Pe [1]	Eq. (2)	$\frac{d_s}{D}=2Fr\,\mathrm{sech}(1.7\frac{e}{D})$

Empirical prediction equations in studies of hydraulic and scour models could be analysed using statistical indices to verify the agreement between observed and predicted data [12,13]. The correlation coefficient (R), root mean square error (RMSE), mean absolute percentage error (MAPE), and scatter index (SI) were applied in this study to analyze scour results below the pipeline under river flows. The parameters R, RMSE, MAPE, and SI can be calculated from Eqs. (6), (7), (8), and (9), respectively [14-17].

$$R=\frac{\sum_{i=1}^{n}(X_{(Obs.)i}-\overline{X_{(Obs.)i}})(X_{(Pre.)i}-\overline{X_{(Pre.)}})}{\sqrt{\sum_{i=1}^{n}(X_{(Obs.)i}-\overline{X_{(Obs.)i}})^2\sum_{i=1}^{n}(X_{(Pre.)i}-\overline{X_{(Pre.)i}})^2}} \quad (6)$$

$$RMSE=\sqrt{\frac{\sqrt{\sum_{i=1}^{n}(X_{(Obs.)i}-X_{(Pre.)i})^2}}{n}} \quad (7)$$

$$MAPE\%=\frac{100}{n}\sum_{i=1}^{n}\left|\frac{X_{(Obs.)i}-X_{(Pre.)i}}{X_{(Obs.)i}}\right| \quad (8)$$

$$SI=\frac{RMSE}{\overline{X_{(Obs.)}}} \quad (9)$$

In the above equations, n is the number of data, $X_{(Obs.)\,i}$ is the observed data, $X_{(Pre.)\,i}$ is the predicted data, $\overline{X_{(Obs.)}}$ is the average of the observed data, and $\overline{X_{(Pre.)}}$ is the average of the predicted data.

3. ***RESULTS AND DISCUSSION***

Fig. 1 provides the predicted data for dimensionless maximum scour depth calculated using an equation of Kjeldsen et al. [9] using 310 observed data from studies of Moncada-M and Aguirre-Pe [1], Hosseini et al.

[3], Dey and Singh [4], Haj-Mohammadi et al. [7], and Pourmirza and Kamanbedast [8]. This empirical equation can predict d_s/D with R=0.327, RMSE=0.384, MAPE=106.01%, and SI=0.409.

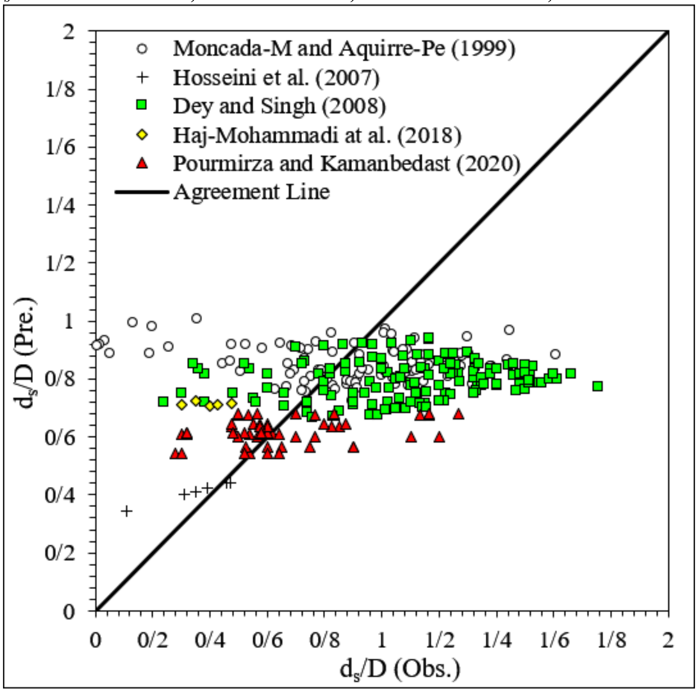

Fig. 1. *Calculation of Maximum scour depth via an equation of Kjeldsen et al. [9]*

The predicted values for dimensionless maximum scour depth from an equation of Bijker and Leeuwestein [10] are presented in Fig. 2 using 310 observed data from studies of Moncada-M and Aguirre-Pe [1], Hosseini et al. [3], Dey and Singh [4], Haj-Mohammadi et al. [7], and Pourmirza and Kamanbedast [8]. Eq. (4) predicts d_s/D with R=0.355, RMSE=0.356, MAPE=131.77%, and SI=0.379.

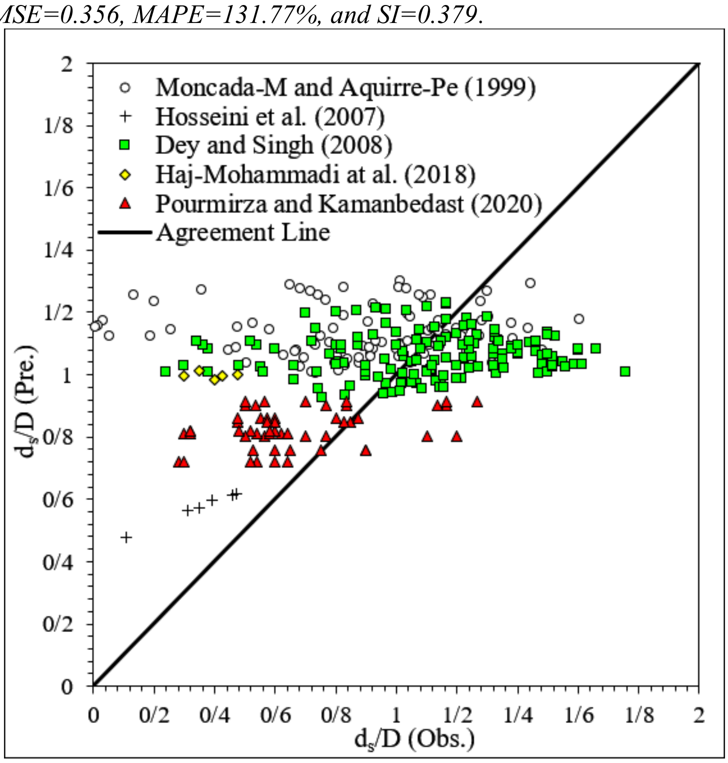

Fig. 2. *Calculation of Maximum scour depth via an equation of Bijker and Leeuwestein [10]*

The estimated values for dimensionless maximum scour depth via an equation of Ibrahim and Nalluri [11] are represented in Fig. 3 using 160 observed data from studies of Dey and Singh [4], and Haj-Mohammadi et al. [7]. Eq. (5) calculates d_s/D with R=0.048, RMSE=0.637, MAPE=50.46%, and SI=0.585.

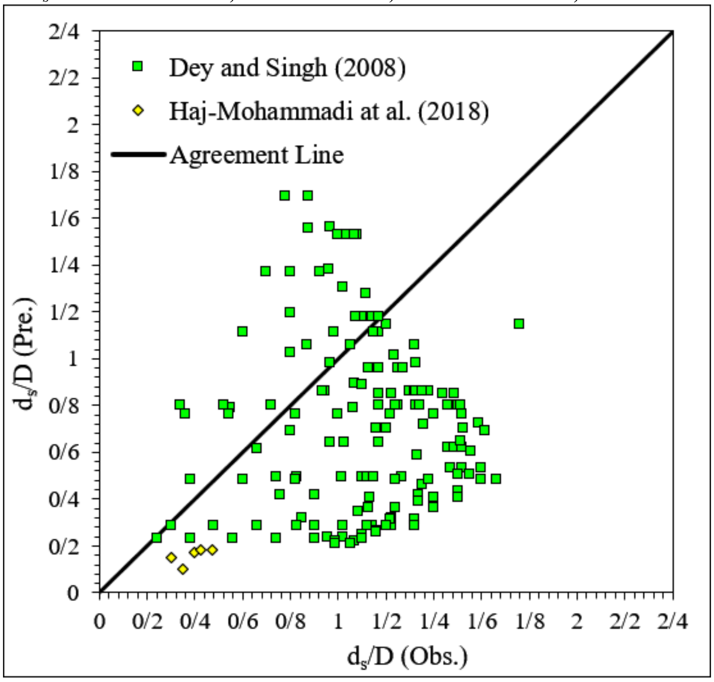

Fig. 3. *Calculation of Maximum scour depth via an equation of Ibrahim and Nalluri [11]*

Fig. 4 presents the predicted values for dimensionless maximum scour depth with an equation of Moncada-M and Aguirre-Pe [1] (Eq. (1)), using 310 observed data from studies of Moncada-M and Aguirre-Pe [1], Hosseini et al. [3], Dey and Singh [4], Haj-Mohammadi et al. [7], and Pourmirza and Kamanbedast [8]. This equation predicts d_s/D with R=0.33, RMSE=0.35, MAPE=119.41%, and SI=0.373.

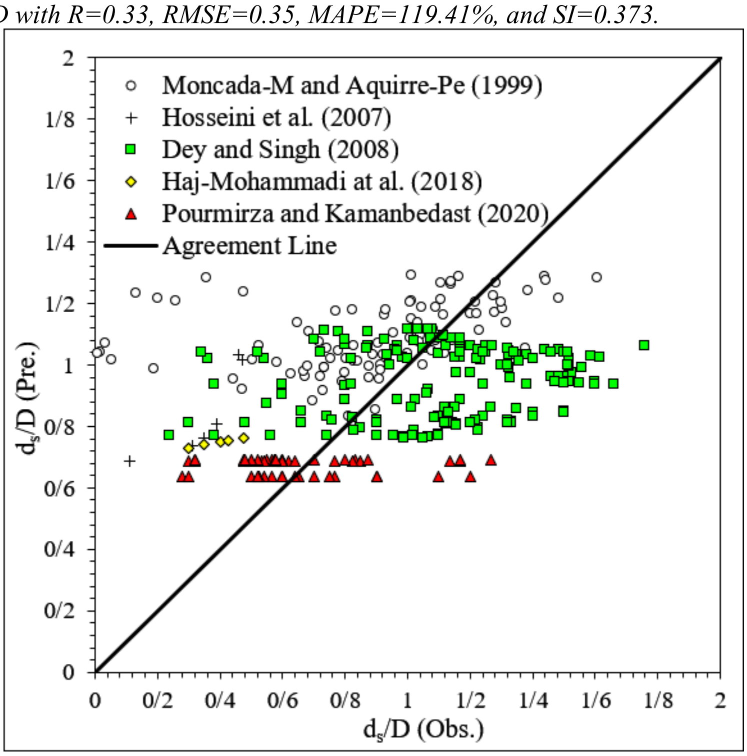

Fig. 4. *Calculation of Maximum scour depth via an equation of Moncada-M and Aguirre-Pe [1] (Eq. (1))*

Fig. 5 shows the estimated values for dimensionless maximum scour depth with an equation of Moncada-M and Aguirre-Pe [1] (Eq. (2)), using 310 observed data from studies of Moncada-M and Aguirre-Pe [1], Hosseini et al. [3], Dey and Singh [4], Haj-Mohammadi et al. [7], and Pourmirza and Kamanbedast [8]. Eq. (2) predicts d_s/D with R=0.4, RMSE=0.478, MAPE=83.19%, and SI=0.718.

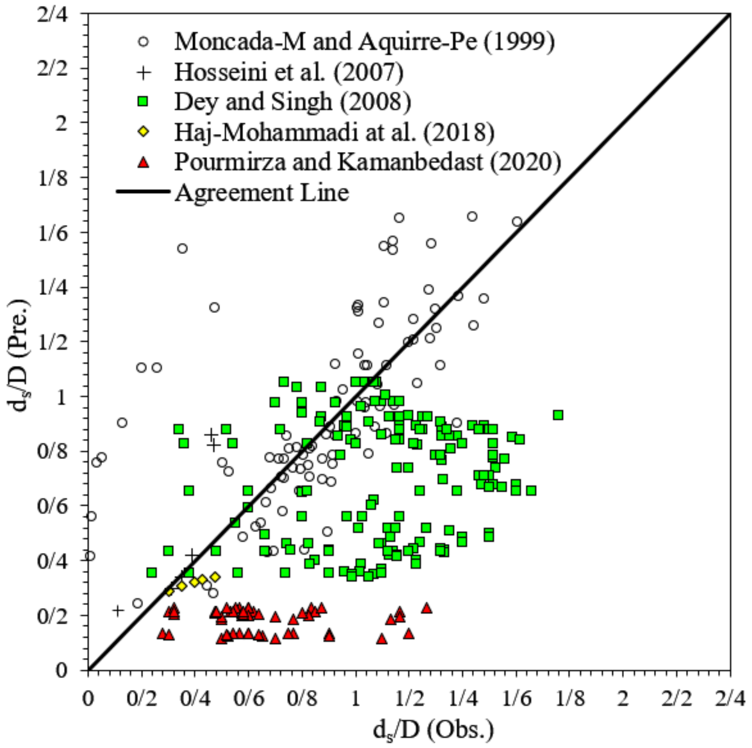

Fig. 5. *Calculation of Maximum scour depth via an equation of Moncada-M and Aguirre-Pe [1] (Eq. (2))*

The modified equations were developed to improve the accuracy of traditional empirical equations using the collected data to predict the maximum dimensionless scour hole depth under the pipeline under flow. Table 3 shows these modified equations along with the calculated statistical index values.

Table 3. *Statistical indices of modified equations for computation of maximum scour depth*

Equation No.	Modified Equation	R	RMSE	MAPE (%)	SI
Eq. (10)	$\frac{d_s}{D} = 1.09\left(\frac{V_0}{2gD}\right)^{0.161}$	0.333	0.346	122.15	0.369
Eq. (11)	$\frac{d_s}{D} = 4.872\left(\frac{V_0}{2gD}\right)^{0.402} D^{0.248} d_{50}^{-0.118}$	0.584	0.298	91.13	0.317
Eq. (12)	$\frac{d_s}{D} = 1.184\left(\frac{V_0}{V_c}\right)^{2.998}\left(\frac{V_0}{gy_0}\right)^{0.058} + 0.314$	0.386	0.219	15.45	0.201
Eq. (13)	$\frac{d_s}{D} = \tanh(0.726Fr) + 0.695$	0.313	0.348	117.56	0.371
Eq. (14)	$\frac{d_s}{D} = 2.503Fr\,\mathrm{sech}\left(-3.228\frac{e}{D}\right)$	0.479	0.443	66.44	0.472

Eq. (10) with R=0.333, RMSE=0.346, MAPE=122.15%, and SI=0.369 improved the accuracy of an equation of Kjeldsen et al. [9] with R=0.327, RMSE=0.384, MAPE=106.01%, and SI=0.409. Eq. (11) increased the accuracy of Bijker and Leeuwestein [10] with R, RMSE, MAPE, and SI from 0.355, 0.356, 131.77%, and 0.379. to 0.584, 0.298, 91.13, and 0.317, respectively. Eq. (12) improved an equation of Ibrahim and Nalluri [11] with R=0.048, RMSE=0.637, MAPE=50.46%, and SI=0.585 to R=0.386, RMSE=0.219, MAPE=15.45%, and SI=0.201. Eq. (13) improved an equation of Moncada-M and Aguirre-Pe [1] (Eq. (1)) with R=0.33, RMSE=0.35, MAPE=119.41%, and SI=0.373 to R=0.313, RMSE=0.348, MAPE=117.56%, and SI=0.371. Eq. (14) provided the modified form of an equation of Moncada-M and Aguirre-Pe [1] (Eq. (2)) with R=0.4, RMSE=0.478, MAPE=83.19%, and SI=0.718 to R=0.479, RMSE=0.443, MAPE=66.44%, and SI=0.472.

4. *CONCLUSIONS*

This study investigated the accuracy of empirical equations for predicting the maximum scour depth under a single pipeline using statistical indices and experimental data. The results show that the equation from the study of Kjeldsen et al. [9] predicts maximum scour depth with R=0.327, RMSE=0.384, MAPE=106.01%, and SI=0.409. The accuracy of this equation was improved by the new equation with R=0.333, RMSE=0.346, MAPE=122.15%, and SI=0.369. The suggested equation of Bijker and Leeuwestein [10] predicts maximum scour depth with R, RMSE, MAPE, and SI from 0.355, 0.356, 131.77%, and 0.379, while the modified equation has R=0.584, RMSE=0.298, MAPE=91.13, and SI=0.317. The equation of Ibrahim and Nalluri [11] has R=0.048, RMSE=0.637, MAPE=50.46%, and SI=0.585 in the prediction of maximum scour depth, and its accuracy was enhanced by a new equation with R=0.386, RMSE=0.219, MAPE=15.45%, and SI=0.201. The equation proposed by Moncada-M and Aguirre-Pe [1] based on Fr has R=0.33, RMSE=0.35, MAPE=119.41%, and SI=0.373, while its modified format has R=0.313, RMSE=0.348, MAPE=117.56%, and SI=0.371 in scour depth prediction. The proposed equation by Moncada-M and Aguirre-Pe [1] based on Fr and e/D, predicts maximum scour depth with R=0.4, RMSE=0.478, MAPE=83.19%, and SI=0.718; while the modified format has R=0.479, RMSE=0.443, MAPE=66.44%, and SI=0.472.

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