Optimizing the dynamic parameters of the vibration equation of motion with the jerk effect – part-I

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ABSTRACT

In common differential equations of the second order of vibration of the mass and spring system, the internal forces include only the effects of displacement and its first and second derivatives, and if there are higher order derivatives, these effects should be considered. In the type of loading and in the physical structure of mass and spring, the reasons for the existence of this force are presented, and by solving the 3rd order differential equation of motion in the state of free vibration, limit points have been determined. The compliance of the structural response is shown in the 2nd and 3rd order motion equations and the test results With harmonic loading, the response spectrum of structural forces in the 3rd order equation has been compared with the 2nd order equation. When the ratio of the loading frequency to the natural frequency of the structure is equal to 3.67 the jerk force will be equal to the damping force. With the increase of loading frequency in large damping ratios, the jerk force tends to the amplitude of the harmonic loading force and other forces decrease, which indicates the intensity of this force in loading with large frequencies. Therefore, in high-frequency earthquakes, in high-rise structures or structures in non-linear behavior that have a long period of time, with the increase of the damping force, the contribution of the jerk force in the balance of internal forces will increase.

Keywords: *Jerk, Optimization, Dynamic equilibrium, Damping, Resonance, Jerk coefficient*

1. INTRODUCTION

In dynamic equilibrium equations, internal forces caused by displacement, velocity and acceleration are considered, but the necessity of other forces caused by derivatives higher than acceleration are not considered in the dynamic equation. First, the logical explanation of the necessity of the existence of those forces is discussed, and the mathematical equations governing the problem and the results of it are presented. Based on the basic scientific research method, it is necessary to conduct experimental tests, and this article first deal with the mathematical relationships between internal forces in the equilibrium of the system. In his fourth philosophical rule, Newton stated that in empirical philosophy, we come across propositions that have been deduced from phenomena by the method of general induction and with great accuracy, and as long as other phenomena have not occurred that make these propositions more accurate or subject to exceptions, we do not accept other conceivable and contrary assumptions[1, 2] . Newton's first philosophical law states that nature does nothing in vain. Therefore, when the phenomenon of an earthquake is accompanied by changes more than acceleration or displacement changes with respect to time in higher orders, to describe their non-usefulness and their effects, to explain the real causes and the effect of those changes on the balance of the internal forces *of the structure, research should be done. Newton's fourth definition states that a body maintains any new state it acquires only by its inertia . In this article, it is assumed that if it is possible to generalize this inertial in the momentum (p), caused by the speed, to movements with higher derivatives, it can be defined in the new state as acceleration. In this case, this action should be considered in the structure's vibration equation in case of ground vibration with variable acceleration.*

2. PROBLEM STATEMENT

The forces acting on objects are defined as surface or body, which are also considered internal or external to the object. To define the concept of stiffness, when we put a rod under tension in the laboratory, between stress and strain, E number or elasticity coefficient is measured. Regardless of the numerical value of E, there is a philosophical concept between the cause (stress) and the effect (strain) that expresses the effect's need for the cause, whose numerical value is obtained experimentally. This issue is not only from a philosophical point of view, but also from the point of view of understanding the relationship between stress and strain and then measuring and generalizing it. By develop this relationship to force-displacement, the concept of hardness K emerges, whose numerical value is measured in axial members by E, A, L and in bending members by E, I and L . This concept in Newton's laws of acceleration by stating the second law (the third definition in the book of Mathematical Principles of Natural Philosophy) , leads to the explanation of the relationship between the cause (force) and the effect (change in the amount of movement or acceleration), and the constant coefficient for this relationship, i.e. the mass m, is measured. Although the inertial mass is equivalent to the gravitational mass, the inertial mass m is defined by the acceleration and is only a proportionality coefficient between the inertial force and the acceleration of the object. The development of this concept leads to the prediction of the relationship between force and speed in the structure with the damping coefficient C, which estimates the value of this constant with laboratory and energy methods. These concepts are presented by integrating the internal forces of the structures with the dynamic equilibrium equation of the single degree of freedom system as follows:

$$F_I + F_C + F_S = P(t) \qquad (1)$$

$$m\,\ddot{u} + c\,\dot{u} + k\,u = p(t) \qquad (2)$$

and therefore, if there are other causes based on higher derivatives of displacement, the equality of the internal force with the external force can be generalized as follows:

$$\sum_{i=0}^{n} a_i \frac{d^i u}{dt^i} = p(t) \qquad (3)$$

The values of a_i have been calculated so far for i equal to zero, 1 and 2, called k, m, c, and they must be calculated, if there are derivatives of higher order for displacement, such as the third derivative, that is, jerk. Considering that only two initial conditions u_0 and $\dot{u}_0$ are needed to solve equation (2), and if the initial acceleration $\ddot{u}_0$ is different but the displacement and initial velocity are the same, then a solution should be found for it, because the initial conditions of displacement and velocity are enough to solve the equation. The solution of equation (2) will be obtained similar to the equation of a line with two primary fixed points. But if there are three primary fixed points that create a curved path with the condition of three points, that curved path with the equation of a straight line with the condition of two fixed points, it cannot be drawn accurately. Therefore, a higher order equation is needed. In this article, a_3 is investigated as the jerk coefficient j. Similar to the stiffness method in the analysis of structures, as the stiffness constant k can be considered as the force required per unit displacement, in which case the damping constant c is also the force required per unit speed, and the mass constant m is equal to the force required to per unit acceleration will be defined. In this way, the jerk coefficient j is defined as the force required for the third derivative of the displacement $\dddot{u}$ or unit jerk, which creates the jerk force Fj, and equation (4) is obtained in dynamic equilibrium.

$$j\,\dddot{u} + m\,\ddot{u} + c\,\dot{u} + k\,u = p(t) \qquad (4)$$

The study of the performance of the jerk force in the resistance of cross-section materials, such as the damping force in ductility, is not within the scope of this article, and this article only deals with the analytical issues of this force. The proposed model of the structure is shown in Figure 1.

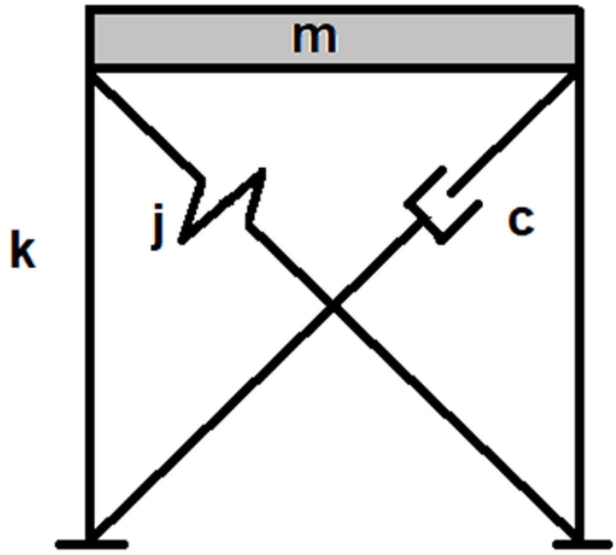

Figure 1- Proposed structural model

3. *The importance and necessity*

So far, the measurement of mass has been done in absolute proportion to the weight. based on the external force such as gravity that creates acceleration, the inertial force is calculated as the internal force of the objects. Stiffness force as the internal force of structures can be measured directly with a specific test or with equations of work or energy caused by external force based on the geometric and physical characteristics of the structure. Laboratory methods can be used to measure damping for any structure in internal force and it is usually expressed as a percentage of critical damping that is proportional to the stiffness and mass of the structure. In other words, like stiffness, damping can be calculated directly from testing and drawing the speed-time diagram or with the help of work and energy equations, in which case equation 2 is also used. Although many tests have been done for concrete, steel and even wood to measure damping[3] ,but analytical equations or experimental diagrams have not been available to calculate the jerk force so far, so that they can determine the value of the jerk coefficient, and secondly, the force caused by the jerk can be investigated in the dynamic equation of motion. To determine how the jerk force exists, by classifying the forces into internal and external in all the components of an object, this force must be classified like the damping force, which is an internal or external surface. Of course, it should be noted that the damping is obtained by equation 2, proportional to m and k, just as the inertial force proportional to the mass m is an internal volumetric force that appears in the acceleration effect. In fact, external forces create internal forces. If external forces cause motion and motion is related to internal forces, it can be expected that changes in motion, i.e. speed, rotation, acceleration and jerk, are related to different internal forces. With this point of view, if we consider gravity as the factor of acceleration and acceleration in relation to inertial force, if even gravity causes a jerk, the jerk coefficient j will also exist in the object like the acceleration coefficient m.

The friction force can be considered as the surface internal elastic force before the movement of the free body, and as the surface external damping force such as Coulomb damping after the movement. The jerk force can be just an internal surface force, i.e. relatively between the center of mass and the support, or it can be considered as a volumetric force, i.e. absolutely at the center of mass, like inertial force. In this case, the calculation of the effective force in the dynamic equation with the vibration of the support, such as an earthquake, can be in one of the following two ways:

$$j\,\ddot{u}_r + m\,\ddot{u}_r + c\,\dot{u}_r + k\,u_r = -j\,\ddot{u}_g - m\,\ddot{u}_g \qquad (5)$$

$$j\,\ddot{u}_r + m\,\ddot{u}_r + c\,\dot{u}_r + k\,u = -m\,\ddot{u}_g \qquad (6)$$

What is important is that the force caused by the jerk $\dddot{u}$ is considered as the jerk force Fj in the equilibrium calculations.

4. Research history

So far, the issue of jerk has been paid attention to from various aspects, including the methods of mathematical solution of the jerk equation, the kinematics of particle movement, the physiological effects of jerk on the human body, geological cases and earthquakes, but the jerk has not yet been investigated as an independent force in the dynamic equilibrium equations. Hayati, H.[4] has provided a complex archive containing 207 references in most scientific fields and their classification, including 207 references in 21 scientific fields such as engineering , health, mathematics and control, since the 17th century. Although she usually did not have the possibility to go into the details in every reference, but as much as possible, she has stated the generalities of those sources for the researchers in the tables. All the sources mentioned in this source are not available, but the topics related to them can be extracted from other related sources. Although apart from these sources, research is expanding every day. The manufacture of jerk gauges is also one of the technological and research developments that patent and electronic research has been done in this field as well[5].

At present, the primary parameters of ground motion measurement and analysis are displacement, velocity, and acceleration, which have been well studied for various seismic source mechanisms, ground motion characteristics, and engineering applications in earthquake engineering. In addition, general or newer parameters such as P-wave energy, S-wave energy[6], seismic moment and location of the earthquake source[7] are very important for ground motions, and precise changes in time domain or frequency domain can be detected. In general, research on the time-frequency characteristics of ground motions and dynamic effects, especially on seismological parameters and earthquake engineering, needs more intensive studies. The jerk record contains a large amount of implicit information about ground motion, which together with other parameters and key responses, research on the jerk and its spectrum can greatly enhance the understanding of seismic instability. In addition, the limitations of some common solution methods in structural dynamics can be better understood and the seismic design philosophy can be improved.

Karmi Mohammadi and et al. [8] studied the performance of the car system and the speed and distance controller with the car in front by using the rules of dynamic car control and by using the gain coefficient, they limited the jerk of the controlled car. They considered a saturation limit for the derivative of vehicle acceleration and deceleration and provided the optimal acceleration for vehicle system performance. This method is used to limit damage caused by jerks in machines. Taghipur, M. et al.[9], by designing an active car suspension system with the aim of simultaneously reducing force and jerk, presented a method to control acceleration and jerk by directly using Hamiltonian instead of Riccati relations, in order to comply with the limit of jerk and its effects on the car body. It is used as a limit point in the objective function. Powel, J. P. et al.[10] was found that both the magnitude of the accelerations and their rate of change (jerk) are important. The results also suggest that there may be scope to improve the trade-off between journey times, energy consumption and passenger comfort by fine control of the acceleration/jerk profile. This is particularly relevant to urban rail systems, as they typically feature relatively high acceleration and deceleration. Rezaeifar, H.[11] proposed a suitable solution for designing the path of robotic arms by designing the path for the robotic arm using midpoints and the minimum jerk method. This method is also based on the kinematic analysis of speed, acceleration and jerk at nodal points.

In 2018 by examining the jerk spectrum of structures and using the derivatives of the sinusoidal function of the structure's movement, similar to the spectral acceleration, Taushanov[12] presented a relationship for the spectral jerk and drew the jerk spectrum of the structure's response. He showed that the impact of the jerk in the near field is greater than that in the far field due to the impact pulse. He also suggested that the effect of the jerk spectrum should be considered in structural designs and the effects of jerk in higher modes need more investigation. Although this method uses the frequency of the structure, it uses the maximum acceleration to calculate the maximum jerk. Taushanov [13], by examining the composite response spectrum curve for a number of map accelerations and the effect of damping on them, introduced a new variable called Miami, which evaluated the effect of the magnitude of earthquakes. He concluded that the jerk response spectrum *follows the same rules as other spectra and the maximum jerk amplitude effect occurs in structures with medium and short cycle time.*

In geophysics, a geomagnetic jerk or geomagnetic shock-like variation is a relatively sudden change in the second derivative of the Earth's magnetic field with respect to time. Such occurrences were noted by Curtillo et al. in 1978, [14] The clearest of them have been observed in 1969, 1978, and 1991 across the globe, and geologists believe that this phenomenon originates from within the earth, but their exact cause is still under investigation. An, Y. et al. [15] proposed the jerk energy parameter, similar to the acceleration energy, which was the product of the power of two accelerations in the mass, and then presented the resulting relations and then the results of applying the acceleration maps on a structure with six degrees of freedom in a laboratory. He showed that the test and analysis results are in good agreement with structural failures and the use of jerk energy variable can be used by engineers.

Ozen [16] presented the mathematical relationship of the jerk vector in a curved and cylindrical path with the matrix method, which this study can be used in terms of kinematics in the movement of particles and even the three-dimensional movement of the earth in the accelerograms.

Among the interesting studies on the subject of jerk, we can refer to the research of Eichhorn[17] in the field of solving jerk equations. By examining the types of differential equations of the third order that are also used in the field of dynamics, he showed the chaotic characteristics of these mathematical equations and categorized them based on the intensity of disturbance. He presented a criterion that in some of these equations, the confusion of their explicit answer can be limited, and the equations used in this article are included in that category.

5. Analysis

The equations of motion governing the dynamic displacement of systems are commonly obtained from three methods: D'Alembert's principle, Hamilton's principle and virtual work principle. The first method was presented with relations (5) and (6). Hamilton's method is based on the numerical quantities of kinetic energy and potential energy and is in the form of equation (7):

$$\int_{t_1}^{t_2} d(T-V)dt \; + \; \int_{t_1}^{t_2} dW\, dt \; = 0 \qquad (7)$$

T is the kinetic energy and V is the potential energy of the device and W is the work of non-potential forces such as damping or arbitrary external loads. According to this principle, the sum of changes in the kinetic energy and potential of the device and the work of non-potential forces in the time interval from t1 to t2 is equal to zero. In the method of virtual displacements, if small displacements are applied to external forces in equilibrium, which are kinematically acceptable, the work done by all forces is equal to zero. In this method, we determine all incoming forces at the center of mass, which also include inertial forces. Due to the presence of the jerk force in the dynamic balance equations, the definitions in the above methods will be more generalized than before. The elastic force is limited to small deformations caused by the displacement of the structure and is related to the constant k, and the amount of energy stored in it per displacement u is equal to:

$$V = \frac{1}{2}ku^2 \qquad (8)$$

If the damping force is related to the velocity by the proportionality factor c, this force has been considered relative between the velocity of the center of mass and the support, and the resulting work is as follows:

$$W = -C\,\dot{u}du \qquad (9)$$

For the jerk force, according to equations (5) and (6) and the unknown nature of the body's behavior against the acceleration derivative, it can be assumed either kinetic energy or non-potential work. The possibility of either of these two states, such as the damping force, depends on the test for the force versus the derivative of the acceleration. The kinetic energy and work due to the jerk force are given in equation 10 or 11:

$$T = \int_{x1}^{x2} F \, . dr = \int_{x1}^{x2} (F_I + F_J) \, . dr = \int_{t1}^{t2} mv \, dv \; + \int_{x1}^{x2} j \, \ddot{u} \, dr \qquad (10)$$

$$W = -j \, \ddot{u} \, du \qquad (11)$$

To solve equation 4, we first check it in the free vibration mode with the following variable change method:

$$u(t) = a \, e^{\alpha t} \qquad (12)$$

By using it, the 3rd order characteristic equation will be obtained:

$$j\alpha^3 + m\alpha^2 + c\alpha + k = 0 \qquad (13)$$

The solution of this equation has three roots, and if the value of Δ in the following equation is equal to zero, it has three real roots with one double root:

$$u_1 = 1 \quad , \; u_2 = \frac{-1+i\sqrt{3}}{2} \quad , \quad u_3 = \frac{-1-i\sqrt{3}}{2} \qquad \alpha_k = \frac{-1}{3j}\left(m + u_k \, C + \frac{\Delta_0}{u_k \, c}\right) \qquad (14)$$

$$C = \sqrt[3]{\Delta_1 + \sqrt{\Delta_1^2 - 4\Delta_0^3}}$$

$$\Delta_0 = m^2 - 3\,j\,c \, , \Delta_1 = 2\,m^3 - 9\,j\,m\,c + 27\,j^2\,k \; , \Delta = 18jmck - 4m^3\,k + m^2\,c^2 - 4j\,c^3 - 27\,j^2\,k^2$$
(15)

Based on whether Δ is greater than, equal to or less than zero, this equation will have three real roots, three real roots of which one root is double, and finally one real root with two complex roots. By solving the Δ equation along with the critical damping obtained from equation 2, the value of the critical jerk coefficient is obtained:

$$j_{cr} = \frac{2}{27}\frac{c\,m}{k} = \frac{4}{27}\frac{m}{w} \quad , \quad c_{cr} = 2\,m\,w \quad , \qquad j_1 = \frac{c\,m}{k} \qquad (16)$$

If the damping value c is chosen to be zero, then Δ will be less than zero, because all the constants are positive and the value of the radical will be negative. In this case, solving Δ will lead to the following equation:

$$j = \pm\, i\frac{m}{w} \times \frac{2\sqrt{3}}{9} \qquad (17)$$

Such a jerk coefficient is not in the domain of real numbers, and in the 3rd order equation, the value of c cannot be equal to zero. By choosing a jerk coefficient smaller than proportional jerk j_{cr}, the displacement will have an imaginary component, which means that the minimum value of the jerk coefficient is the critical jerk. The real value of the displacement also gradually decreases with the reduction of the jerk coefficient to zero, and corresponds to the displacement in the 2nd order equation, although this difference and reduction is very small and does not fluctuate. If the damping is smaller than the critical damping, the jerk coefficient will also decrease accordingly and the structural fluctuations will be proportional to the 2nd order equation. If the damping ratio is more than the critical damping, the jerk coefficient j will not be less than the proportional critical jerk coefficient j_{cr}, but it can increase up to non-proportional jerk j_1.

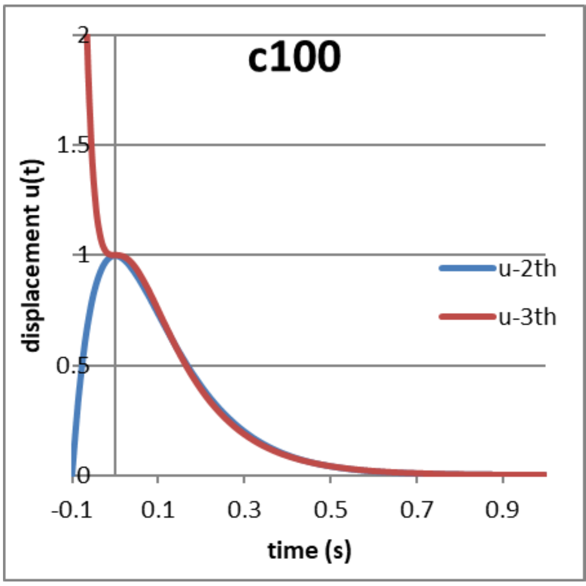

Figure 3- Comparison of 2nd and 3rd order dynamic motion equation with damping and critical jerk with initial condition u_0

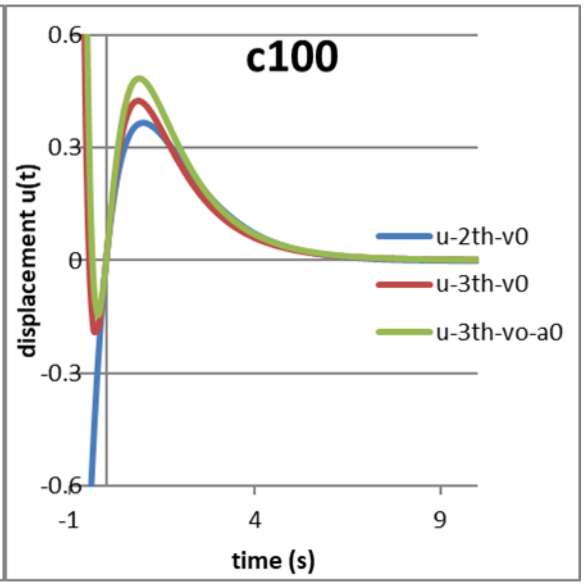

Figure 2- Comparison of 2nd and 3rd order dynamic motion equation with damping and critical jerk with initial velocity $\dot{u}_0$ and initial acceleration $\ddot{u}_0$

In the case that the critical jerk coefficient is used with the condition of critical damping and only with the initial condition of displacement in equation 4, according to Figure 2, the behavior curve of the structure will match the critical state curve of equation 2. But if the initial displacement is equal to zero and the initial conditions of speed and acceleration exist, these two curves will differ according to Figure 3. From the comparison of these two, it can be concluded that in the first case, the damping value obtained from the experiment is equal to the damping value used in equation 2 and 4. But Figure 3 shows that in the second case, even if the damping value is obtained from the test with initial speed and acceleration, it will be different from the value used in equation 2, and in this equation, it is considered more than the actual value. The amount of this difference will depend on the initial speed and acceleration. According to Figure 3, although the curves are different before zero time, but in reality, negative time is not defined. For the value of the non-proportional jerk coefficient j_1 *,while the damping ratio is greater than the critical damping* c_c*, the oscillation of the one-degree-of-freedom system after the initial time of zero continues as a permanent and harmonic oscillation after several cycles. This state is similar to the state without damping in equation 2, with the condition of initial velocity* $\dot{u}_0$*, without change of initial location* u_0 *and without initial acceleration* $\ddot{u}_0$ *, but of course with a smaller amplitude. The oscillatory behavior of the structure at four values for the non-proportional jerk* j_1 *coefficient at the critical damping ratio of 100% is shown in Figure 4. For j more than* j_1*, the displacements of the structure are increasing. In fact,* j_{cr} *and* j_1 *,like zero damping and critical damping, are two states for the jerk coefficient. If we obtain two constants c and j based on the 3rd order equation and two independent constants m and k, these two constants will be dependent on each other, which will be according to equation 18. For the value of the proportional jerk coefficient* j_{cr}*, the displacement of the structure based on different damping states for the 3rd order equation is plotted in Figure 5, which corresponds to the response of the 2nd order equation:*

$$j_{cr} = \frac{2}{27}\frac{c\,m}{k} = \frac{4}{27}\frac{m}{w}\zeta_c \quad ، \quad \zeta_c = \frac{c}{c_{cr}} = \frac{c}{2mw} \qquad (18)$$

$$\frac{4}{27}\frac{\zeta_c}{\omega}\,\dddot{u}_r + \ddot{u}_r + 2\omega\,\zeta_c\,\dot{u}_r + \omega\,u = -\ddot{u}_g \qquad (19)$$

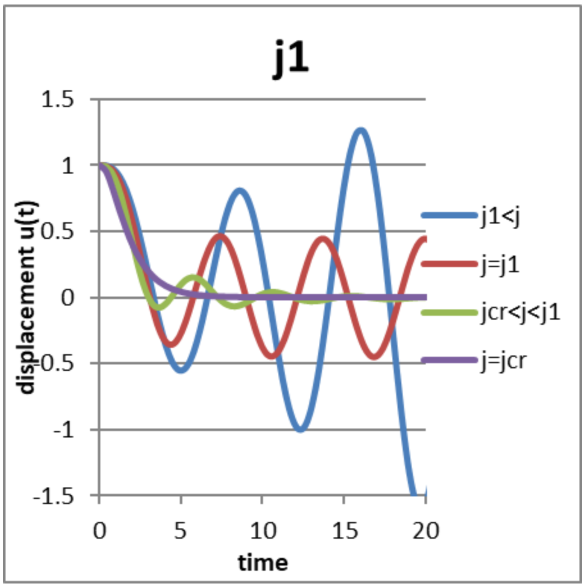

Figure 4- Comparison of system behavior with non-proportional jerk coefficient j_1

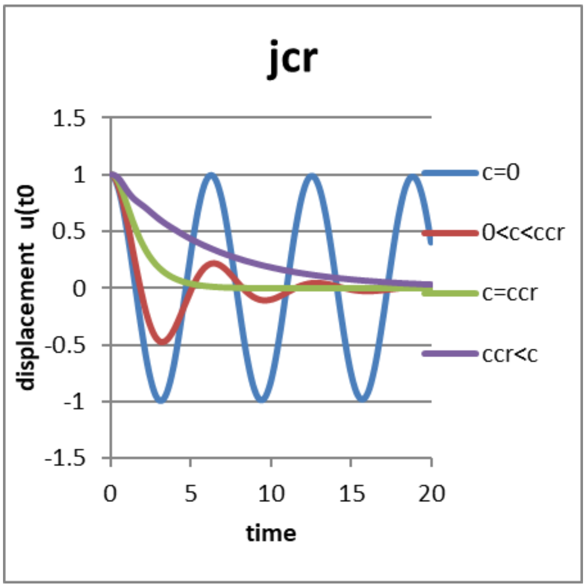

Figure 4- Comparison of system behavior with proportional jerk coefficient j_{cr}

6. *Experiment*

The RD85 laboratory device and the corresponding physical model have been used for this research, Figure 6 .The values of k and m are equal to 32.73 kg/cm and 2.375 kg s2/m, respectively, and LD, Lm, Ls are also equal to 33, 47 and 77 cm. Two modes without damper and with damper have been tested in free vibration with initial displacement of 1 cm. By using the test curve and applying the exponential response of equation 2 by the logarithmic reduction method, the damping ratio of 1.3% for no damper, i.e. the inherent damping of the mass and spring system, and the damping ratio of 12% for the damper has been obtained. The damping correction factor for the 3rd order equation for matching two curves in damping ratios greater than 10% is almost equal to one. The test results and the vibration curve of the 2nd and 3rd order equations using c_j *show a good match, Figu re 7. Therefore, considering that the proportional jerk coefficient* j_{cr} *in at least four points, i.e. damping ratios of 0%, 1.5%, 12% and 100%, creates a good match between the 2nd order and 3rd order equation and the experiment, it can be acceptable.*

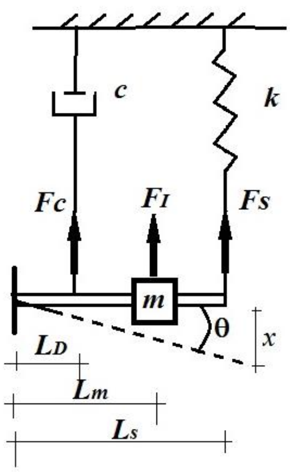

Figure 6- Laboratory device and physical model

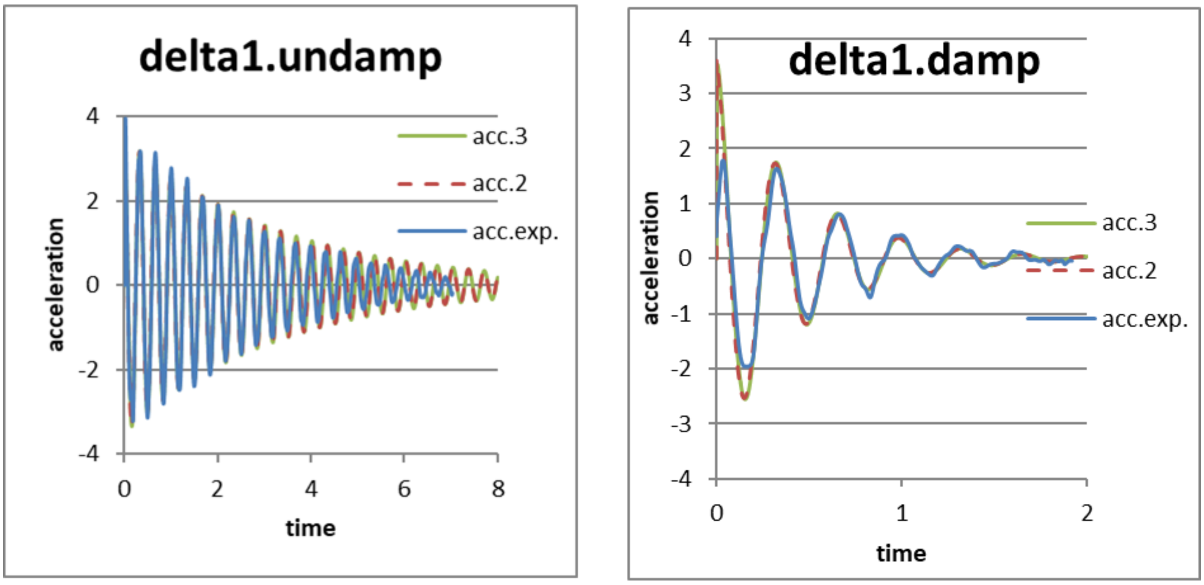

Figure 5-Comparison of displacement of a single degree of freedom structure with and without dampers - 2nd order and 3rd order equation with experiment

7. Discussion , harmonic vibration

The use of harmonic load as a dynamic load is common in connection with the Fourier series, which is able to describe other types of loading such as earthquakes. The dynamic behavior of the structure is a function of both the system's natural frequency and the external loading frequency. The movement due to changes in acceleration causes the jerk force. Assuming that an external force such as gravity, whose acceleration is assumed to be constant, cannot create a jerk force in a free body, but the movements of the earth in an earthquake have variable acceleration and can cause a jerk and a jerk force. The internal forces of the structure also depend on the physical structure of the system, which in the case of mass and spring, by solving the mathematical equation governing the model of mass and spring, the answer will be obtained as a sinusoid, which will have successive derivatives and, as a result, will have a jerk. Therefore, the sinusoidal external force, together with the physical structure, leads to the creation of jerk force in different situations. By introducing a sinusoidal load on a one-degree-of-freedom system with mass m and stiffness k equal to 1 kg.s2/m and 100 kg/m, respectively, and several values of damping c and jerk coefficient j, its behavior has been investigated in equation 20:

$$j\dddot{u} + m\ddot{u} + c\dot{u} \ + ku = \sin\varpi\, t \qquad (20)$$

Assuming that the response frequency of the structure in the steady state is equal to the loading frequency and knowing that in the intensified state, the inertial and stiffness forces are equal and the loading is simply to overcome the damping force, according to equation 21, the ratio of the loading frequency to the natural frequency is equal to one.

$$k\,\sin(\overline{w}t) = m\,\overline{w}^2\,\sin(\overline{w}t) \quad , \quad \beta = \frac{\overline{w}}{w} = 1 \qquad (21)$$

Therefore, for the frequency ratio β where the damping force and the jerk force are equal (Figure 9), we will have:

$$c\ \bar{w}\cos(\bar{w}t) = \left(\frac{2}{27}\ cm/k\right)\bar{w}^3 \cos(\bar{w}t) \quad , \quad \beta = \frac{\bar{w}}{w} = \sqrt{\frac{27}{2}} = 3.67 \qquad (22)$$

The forces fc and fj are plotted in the β ratio equal to 3.67 in the permanent response of a single degree of freedom system with a damping ratio of 30%, Figure 8.

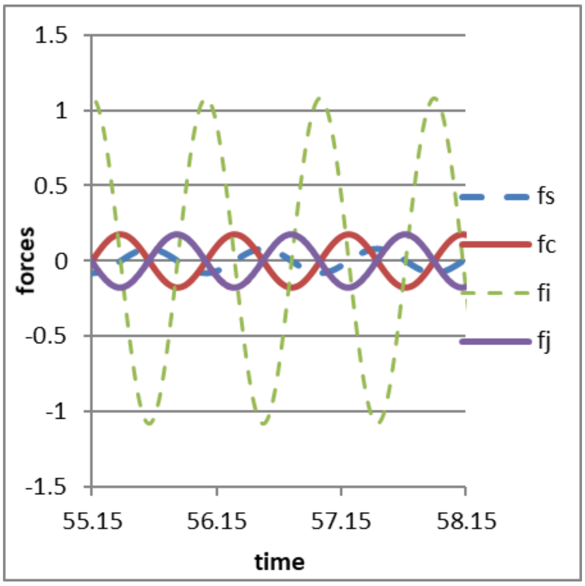

Figure 8- Comparison of fc and fj forces in β ratio equal to 3.67

With the increase of the β ratio from 3.67, the magnitude of the jerk force will be greater than the damping force and will tend towards the maximum harmonic load amplitude force. Inertial force tends to zero like damping force and stiffness force. In the 2nd order equation, there is no jerk force and the inertial force tends to the maximum amplitude of the load. Therefore, the difference between the 2nd and 3rd order equations can be found in large β ratios and proportional to the increase in damping, although the magnitude of these forces is not large compared to the resonance forces and is limited to the maximum loading range, here it is equal to one. Comparison of stiffness, damping and inertia forces in permanent response between 2nd and 3rd order equation in 5% damping in proportional jerk j_{cr} and considering cj does not show any difference. But in the damping ratio of 100% due to the increase of the jerk force, there is a difference between the inertial force and the damping force in the 2nd and 3rd order equation, which shows the effects of the jerk force in high damping, Figure 9.

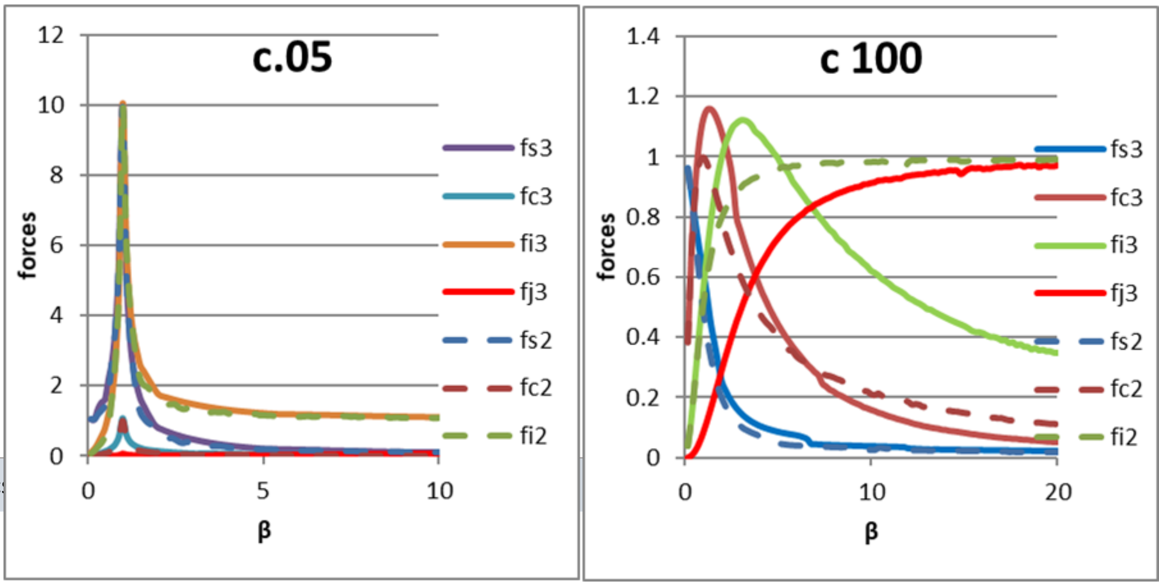

Figure 9-Comparison of stiffness, damping and inertia forces in 2nd and 3rd order equations, damping ratio 5% and 100%

The upward slope of the jerk force in Figure 10 shows that in large damping ratios and the existence of proportional jerk j_{cr}, this force can take large values in high frequency loading or β ratio greater than 3.67, which of course is proportional to the damping ratio. With the increase of damping in β greater than one, although the damping force and inertia force decrease, the jerk force increases.

However, the jerk coefficient is one of the physical properties of materials, such as hardness, damping and mass. Therefore, based on the jerk coefficient, which in other researches has been obtained directly from the experiment based on the jerk-force relationship independently of equation 2 and is not dependent on the damping coefficient with equation 18, it is possible to calculate the jerk coefficient in the non-proportional limit state, despite the ratio Check damping greater than 100%. Due to the development of technology and the use of industrial dampers that meet the needs of the construction industry in dealing with earthquakes, such a jerk will be possible.

With the non-proportional jerk coefficient j1, in the damping ratio greater than the critical damping c_{cr}, in resonance, the damping force and jerk and the stiffness force and inertia are equal to each other, and the damping force is twice as much as the inertia. By increasing the loading frequency from 1 to 50, this ratio is almost maintained, but all the forces have increased up to 50 times, Figure 10. Such a difference caused by the difference in loading frequency was not obtained in the 2nd order equation or the proportional jerk coefficient j_{cr} . Therefore, as long as the test results independent of damping are not obtained, the proportional jerk coefficient j_{cr} can be used as a criterion. ζ

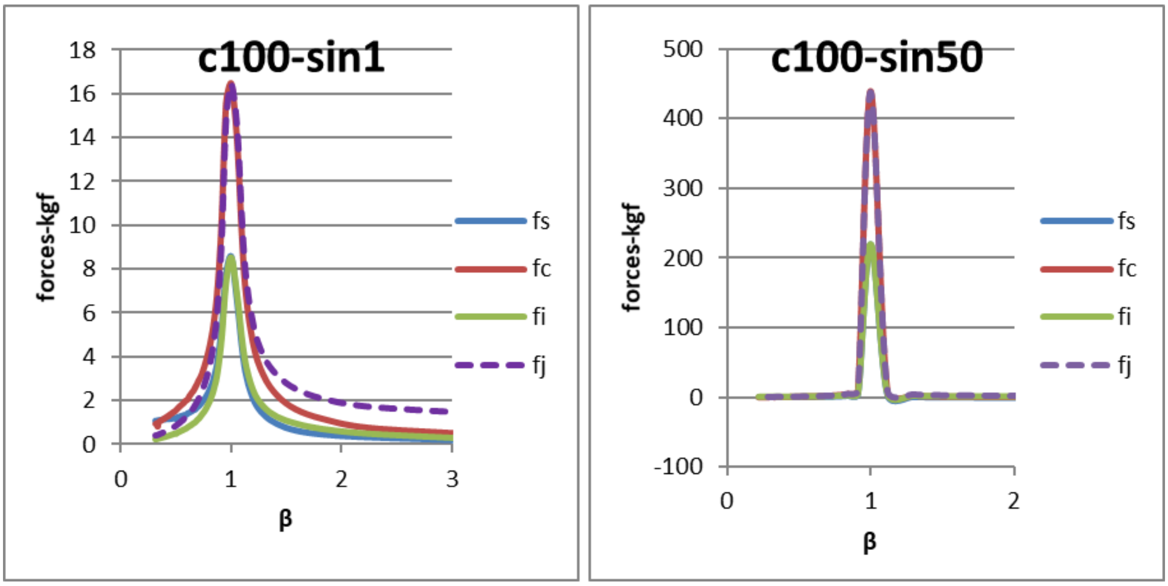

Figure 60-Comparison of stiffness forces, damping, inertia, jerk, damping ratio 100% in sinusoidal load with frequency 1 and 50 rad/s with non-proportionate jerk coefficient j_1

8. Conclusion

The investigation of the forces caused by displacement, velocity and acceleration was done in dynamic equilibrium, and the logical reasons for the necessity of the force caused by the derivative of acceleration were presented and the mathematical equations governing the problem were stated. After showing the situations where this force is important, it suggests the necessity of investigating and determining the contribution of each force in the overall equilibrium of the structure and conducting tests to measure the force in high frequency loading. The equations of motion governing the dynamic displacement of devices were briefly presented by the method of D'Alembert's principle and Hamilton's principle. Solving the equation of motion with the presence of jerk led to two limit states of proportional and non-proportional jerk, which according to the matching of the test curve with the curve of the 3rd order equation, the use of the proportional jerk coefficient is suggested for damping ratio below 100%.

The behavior of the structure in loading with harmonic force showed that considering that the jerk force and the damping force are in opposite phase, in the case that the ratio of the loading frequency to the natural frequency of the system β is greater than 3.67, the jerk force Fj is greater than the damping force. will be. This point shows the intensity of this force in loading with high frequencies and it is necessary to consider the contribution of other forces with the participation of this force.

In large damping ratios, with the increase of β ratio, the jerk force tends to the range of the load force more quickly, and the other forces tend to zero. This point becomes important in high-rise buildings that have a low first mode frequency, against high frequency loading. The same situation in damaged structures whose behavior has entered the range of non-linear behavior and their natural frequency has decreased, can cause the increase in jerk force. Although the damping ratios and the ratio of the jerk coefficient may be small in nature in normal structures, but in case of providing industrial damping or new standards in implementation, these diagrams should be taken into account. For this purpose, the results of this research will be useful for specific laboratory situations.

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